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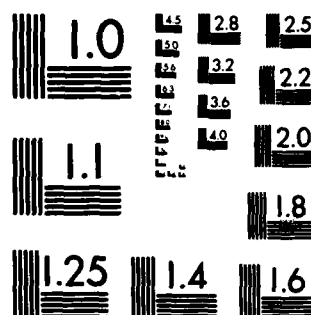
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TRW Space & Technology  
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ANNUAL REPORT

TO THE

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ON

APPLICATIONS OF JOSEPHSON  
JUNCTION SQUIDS AND ARRAYS  
CONTRACT NO. N00014-81-C-0615  
SEPTEMBER 1981 - SEPTEMBER 1982

PREPARED BY

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APPLIED TECHNOLOGY DIVISION  
TRW

NOVEMBER 1982



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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  This is the first Annual Report for an effort to demonstrate the application of SQUIDS (Superconducting QUantum Interference Device) and SQUID arrays to microwave systems. The application considered here is a microwave voltage- controlled oscillator. We report on the circuit design, numerical simulation, experimental design, and fabrication of the device including tests of the monolithic impedance transformer. Predicted available power is approximately 10 nW into a load impedance of 10 ohm.		

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## 1. INTRODUCTION

This is the Annual Progress Report for Contract No. N00014-81-C-0615, "Application of Josephson Junction SQUIDs and Arrays." covering the period from 1 September 1981 through 30 September 1982. The objectives of this program are to define and demonstrate applications of SQUIDs and SQUID arrays via analysis and experiment. The goals of this contract were to investigate and demonstrate the properties of a SQUID voltage-controlled-oscillator (VCO). These goals were met by 1) analysis and simulation of a voltage-clamped resistive SQUID and a voltage-clamped dc SQUID, 2) design, fabrication and measurement of appropriate microwave matching transformer, and, 3) design, fabrication, and measurement of a SQUID VCO. We report on progress in these three areas.

## 2. WORK PLAN

This is a new contract for a projected multi-year investigation of Superconducting QUantum Interference Device (SQUID) arrays and their application to microwave and gigabit technology. The focus of this contract is the investigation and demonstration of a SQUID voltage-controlled oscillator (VCO). The array is then projected to increase the available power and operating impedance. The approach utilizes monolithic superconducting integrated circuits which were designed, fabricated, and tested at TRW.

Three tasks were undertaken in the first year:

Task 1. Analysis of dc SQUID Arrays

Task 2. Design of dc SQUID Generator

Task 3. SQUID VCO Fabrication and Measurement.

A number of the steps are very similar to those required for a contract with the Naval Research Laboratory, "Demonstration of a SQUID Parametric Amplifier." Such steps were carried out jointly and are reported to both agencies. TRW is concurrently conducting Independent Research and Development on Advanced Josephson Devices. This project is principally concerned with development of a Josephson integrated circuit fabrication capability. Some of the activities of that program which impact this contract are discussed here.

### 3. PROGRESS

#### 3.1 Transformer Design

Since the SQUID VCO is necessarily a very low impedance device, a suitable transformer is an essential ingredient for evaluating the basic VCO, particularly with respect to available power. In order to carry out this transformation in both impedance and dimension in a reproducible and predictable manner, the transformer is integrated with the active circuitry.

The basic architecture of the transformer was presented in the contract proposal. We have refined the design for the TRW fabrication process and reconsidered the problems of launching from 50 $\Omega$  coaxial cable into 50 $\Omega$  coplanar line, dimensional step from 1 mm coplanar line at 50 $\Omega$ , and construction of capacitors in the lumped element impedance transformer. The electrical design was optimized numerically using COMPACT with a computed VSWR < 1.25 over the 8-12 GHz band (Figure 1). Figure 2 shows the equivalent electrical circuit and the geometrical layout.

A test of the effect of changing dimensions in 50 $\Omega$  coplanar line was performed by fabricating three 50 $\Omega$  transmission lines: a continuous coplanar line at mm dimensions, a sharply stepped coplanar line, and a short, tapered transition as shown in Figure 3. The measured performance showed no significant differences between the three lines. Therefore, the input LC circuit in the transformer intended to tune out the inductance of the step in the coplanar line was deleted.

The transformer was designed to be fabricated on a 2 inch silicon wafer, 0.015 in. thick, with a minimum line width of 50 $\mu$ m. An OSM microwave launcher drives the 50 $\Omega$  coplanar line which has a center conductor width of 50 $\mu$ m, ground plane spacing of 30 $\mu$ m, and an effective dielectric

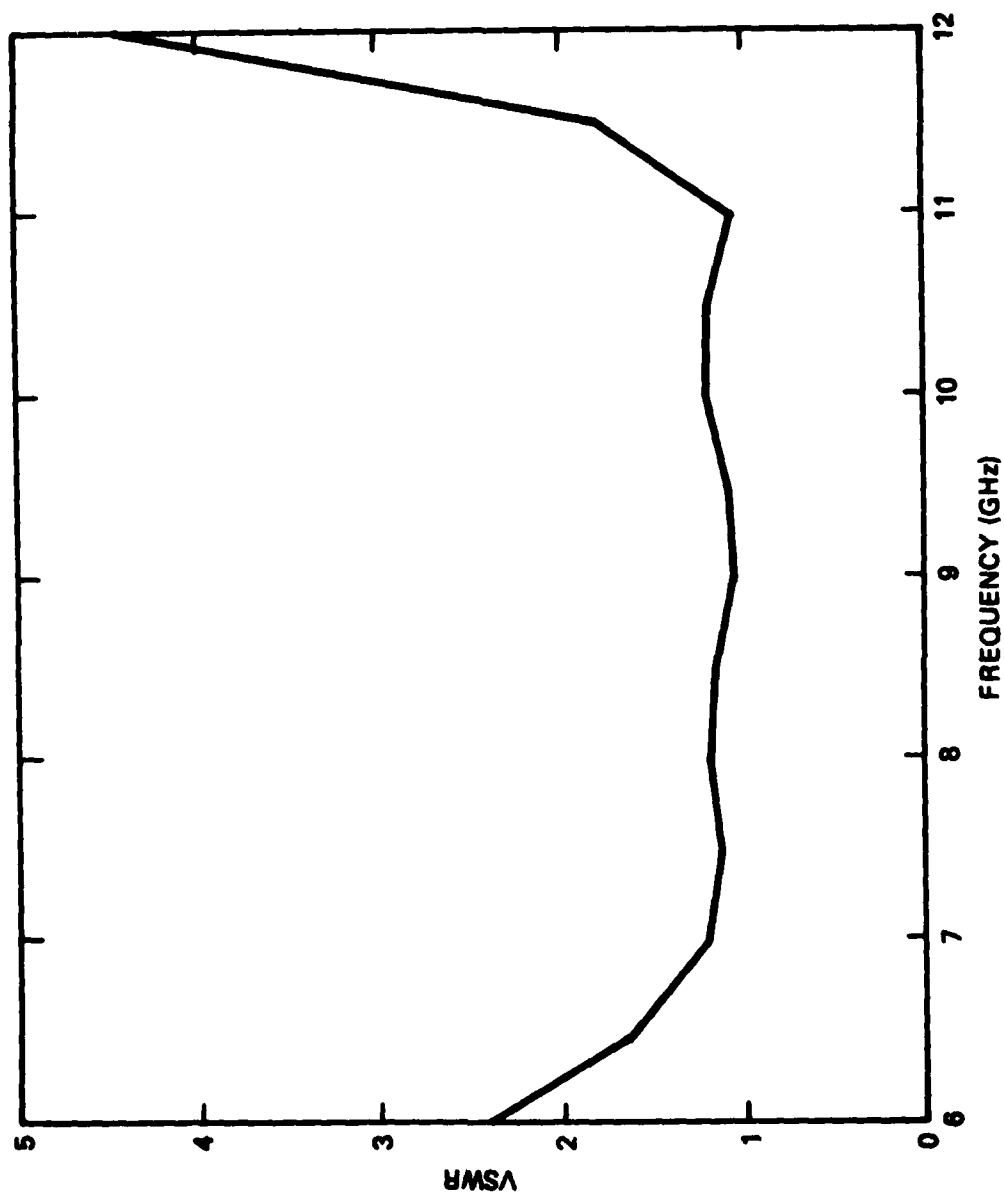


FIGURE 1. Computed voltage standing wave ratio (VSWR) as a function of frequency for transformer circuit shown in Figure 3.

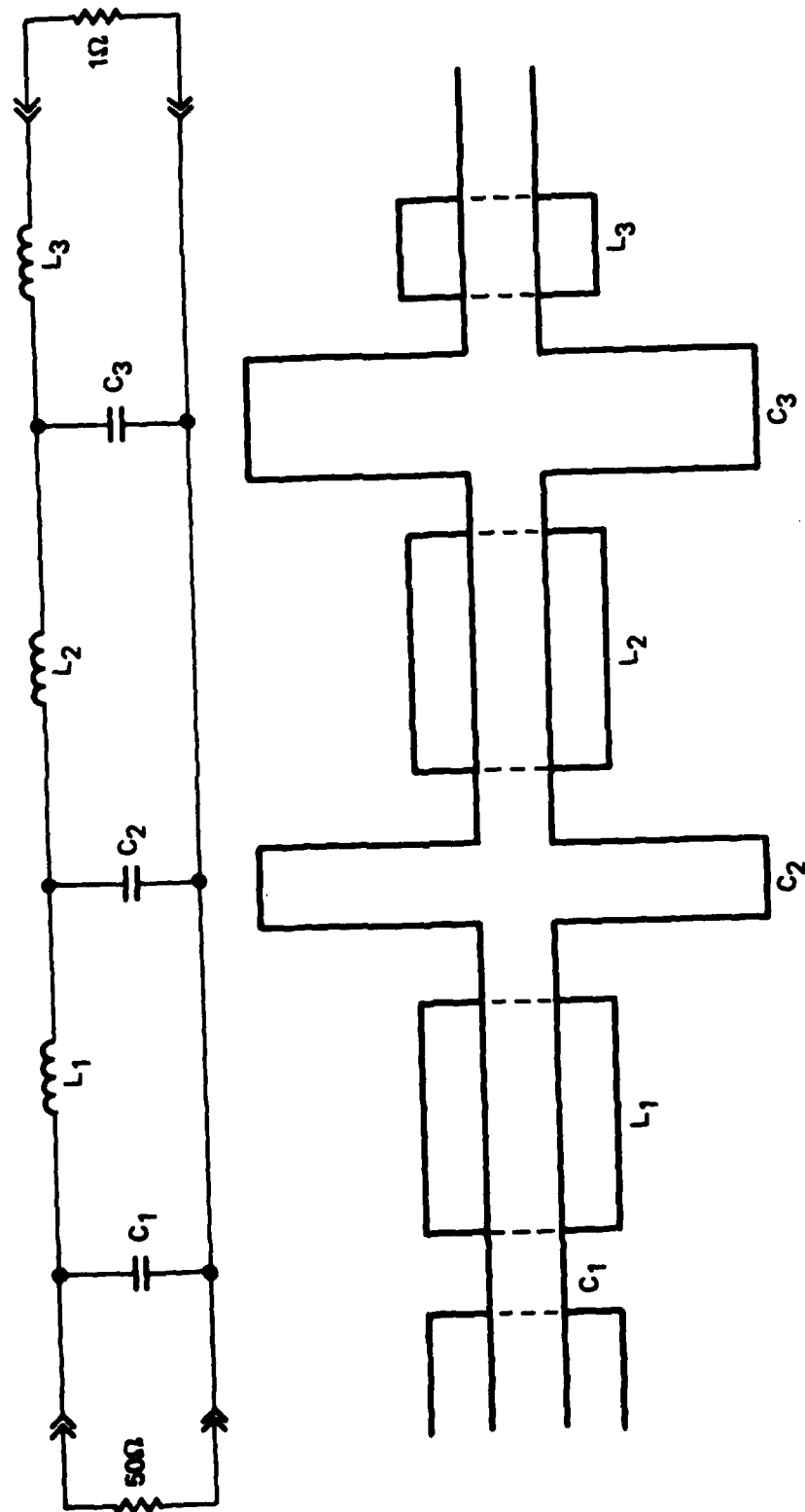


Figure 2. Equivalent circuit (top) and physical layout (not to scale) of X-band transformer designed and simulated. The lumped circuit parameters are  $C_1 = 0.6$  pF,  $L_1 = 442$  pH,  $C_2 = 1.6$  pF,  $L_2 = 108$  pH,  $C_3 = 8.6$  pF,  $L_3 = 22$  pH.

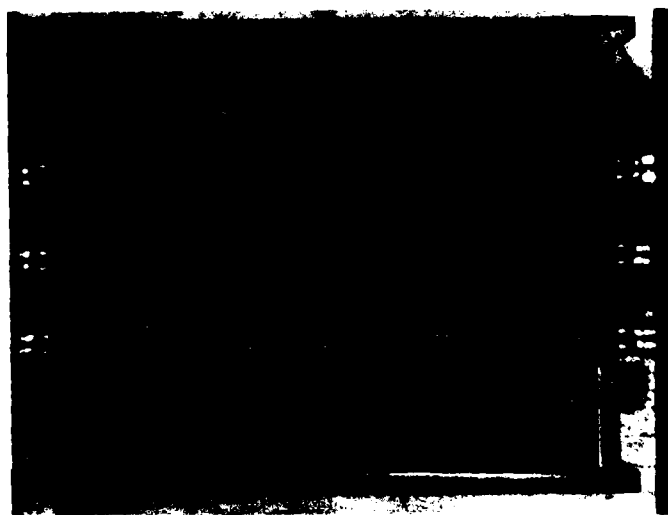


Figure 3. Photograph of the  $50\Omega$  coplanar lines used to test dimensional changes.

constant of 6.4. The reduction in  $\epsilon$  and its dependence on linewidth results from the finite thickness of the Si substrate. The  $1\Omega$  microstrip in the transformer is  $50\mu\text{m}$  wide and separated from the Nb ground plane by  $50\text{nm Nb}_2\text{O}_5$  and  $200\text{nm SiO}_2$ . It has an effective  $\epsilon=13$ .

The transformer design incorporates short ( $< \lambda/4$ ) sections of coplanar and microstrip lines which act as lumped inductors and capacitors, respectively. The inductive lines are  $50\Omega$  coplanar lines terminated by  $1\Omega$  microstrip, with  $L=Z_0\ell/V$ , where  $Z_0$  is the line impedance,  $\ell$  the line length, and  $v$  the propagation velocity. Capacitors are  $1\Omega$  and  $0.5\Omega$  microstrip either terminated by  $50\Omega$  coplanar or used as open parallel stubs. The capacitance of short, open lines is given by  $C=\ell/Z_0v$ .

Photolithographic patterns were defined as 4-level masks and produced at the TRW Microelectronics Center for the combined dimensional and impedance transformer. For the dimensions and tolerances required, masks were fabricated directly at the reticle level in an Electromask pattern generator. Figure 4 shows the computer-generated composite of three  $1\text{ cm} \times 2\text{ cm}$  chips which were fabricated on  $2''$  silicon wafers,  $.015''$  thick. Each chip has 2 coplanar inputs with tapered lines. On one chip the reduced width coplanar line couples directly through; on the second chip, the transformers from each end are connected for in-out transmission measurements. The third chip has each coplanar line and transformer terminated with a matched resistive load. Figure 5 is an expanded plot of the transformer and terminating resistor on the third chip.

The transformers were fabricated in a  $1\text{ cm} \times 2\text{ cm}$  format on  $2''$  diameter silicon wafers with an insulating  $\text{SiO}_2$  surface. Three test chips are produced on each wafer. Following fabrication, the wafers were cleaved with a modified Tempress scribe and the chips mounted in a holder suitable for connection to an OSM coaxial bulkhead launcher. Measurements are made at  $T=4\text{K}$  with an HP-8410 Network Analyzer. The data are in the form of the scattering matrix coefficients  $S_{11}$  and  $S_{12}$ . Only the magnitude of  $S$  is reported since the phase is difficult to calibrate with long coaxial lines into the helium dewar.

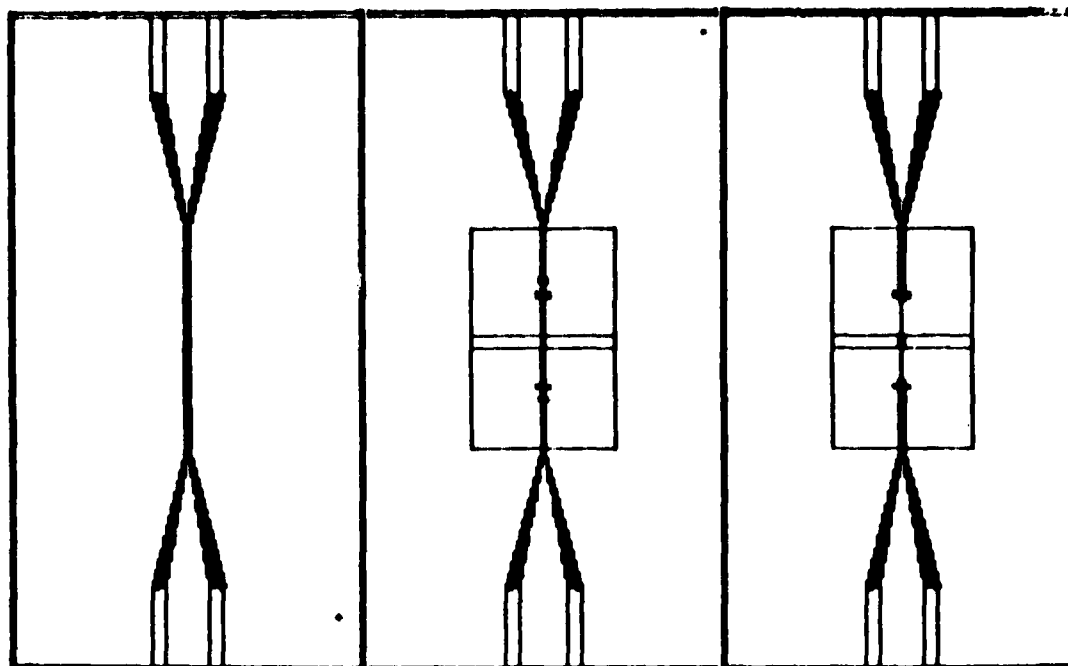


FIGURE 4. Printout of composite masks for the 50 $\Omega$  to 1 $\Omega$  transformers.  
 Top: 50 $\Omega$  stepped coplanar transmission lines; Center: 50 $\Omega$  coplanar to 1 $\Omega$  microstrip to 50 $\Omega$  coplanar line; Bottom: 50 $\Omega$  coplanar to 1 $\Omega$  microstrip terminated.



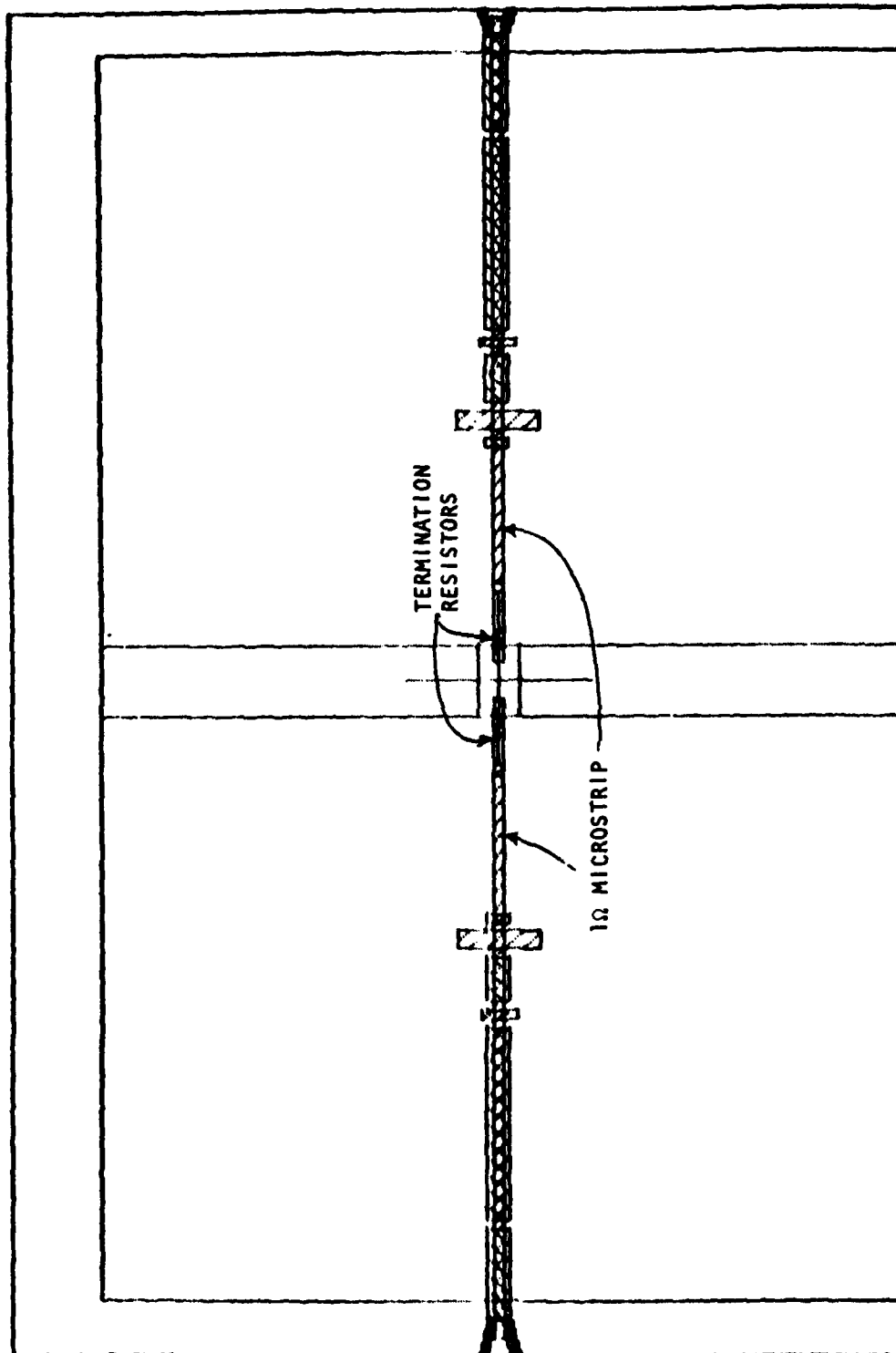


FIGURE 5. Printout of composite masks for the 50Ω to 1Ω transformer terminated in the 1Ω resistor at the center.

WIDE WIRE MMBEP SELECTED 0  
 GATE 5 MC 5 MC OFF 0 MC 0 MC  
 FULL LEVELS 1 2 3 4 . 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

The test transformer chips are mounted with a coaxial connector at each end as shown in Figure 6. As anticipated, connecting the coplanar input to the bulkhead connector was a significant problem. An acceptable solution is to use In solder to connect gold ribbon from the connector centerpin and bulkhead to the coplanar lines. To facilitate soldering to the Nb film, a gold film is evaporated over the Nb. This is done before the Nb is patterned. Figure 7 shows the response of the simple tapered coplanar lines at 4 kelvin; Figure 8 the response of the double-ended transformer, in which the  $50\Omega$  coplanar line is transformed to  $1\Omega$  microstrip at the center of the chip, and then transformed back to  $50\Omega$  coplanar at the other end of the chip. We have not measured good S-parameter characteristics with the terminated transformers and we attribute this to a problem in making good contact with the resistors.

### 3.2 Analysis of Voltage-Clamped SQUIDs

We have investigated the voltage-clamped SQUID in two forms appropriate to the voltage-controlled oscillators, the resistive SQUID of Figure 9 and the dual resistive SQUID with common voltage-biasing resistor which we call the voltage clamped dc SQUID (Figure 10). To the best of our knowledge, neither of these devices incorporating the RSJ model with appropriate capacitance has been fully investigated and thus computer simulation was undertaken to predict the operating characteristics and parameterize the performance.

Analyses were performed with numerical simulations for preselected ranges of the device parameters. Computations of the transient response and subsequent Fourier spectra were performed on a Cyber 174/750 machine. The results are described in a paper which will be given at the 1982 Applied Superconductivity Conference and submitted for publication in the Conference Proceedings to appear in the IEEE Transactions on Magnetism in 1983. The manuscript, "SQUID Voltage-Controlled-Oscillator," is attached as a part of this report. (Appendix A).

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FIGURE 6. Photograph of transformer test chip mounted in a brass holder and connected to two OSM bulkhead connectors. The tapered coplanar lines at each end are visible in the photograph. The grey rectangle in the center of the chip is anodized Nb covered with  $\text{SiO}_2$ . The transformers are located in the same rectangle but are not clearly visible in the photograph.

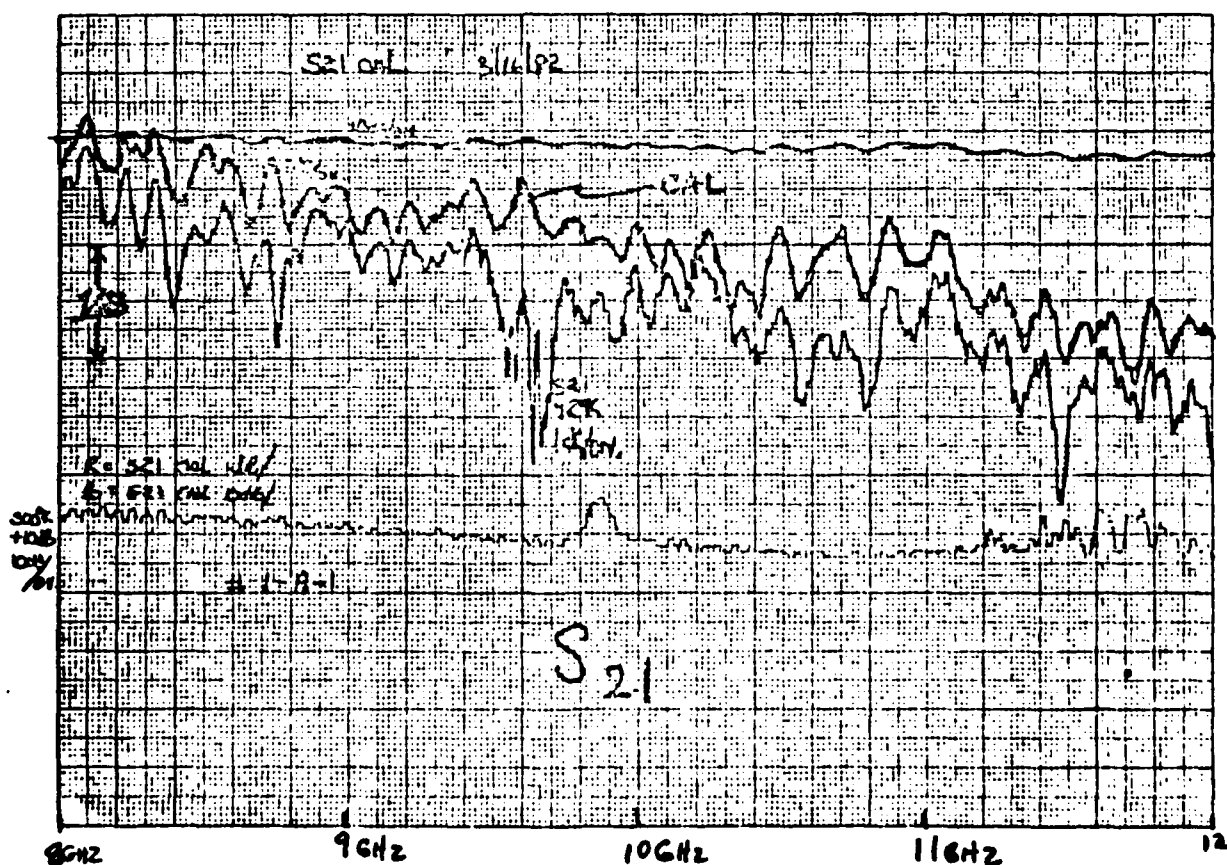
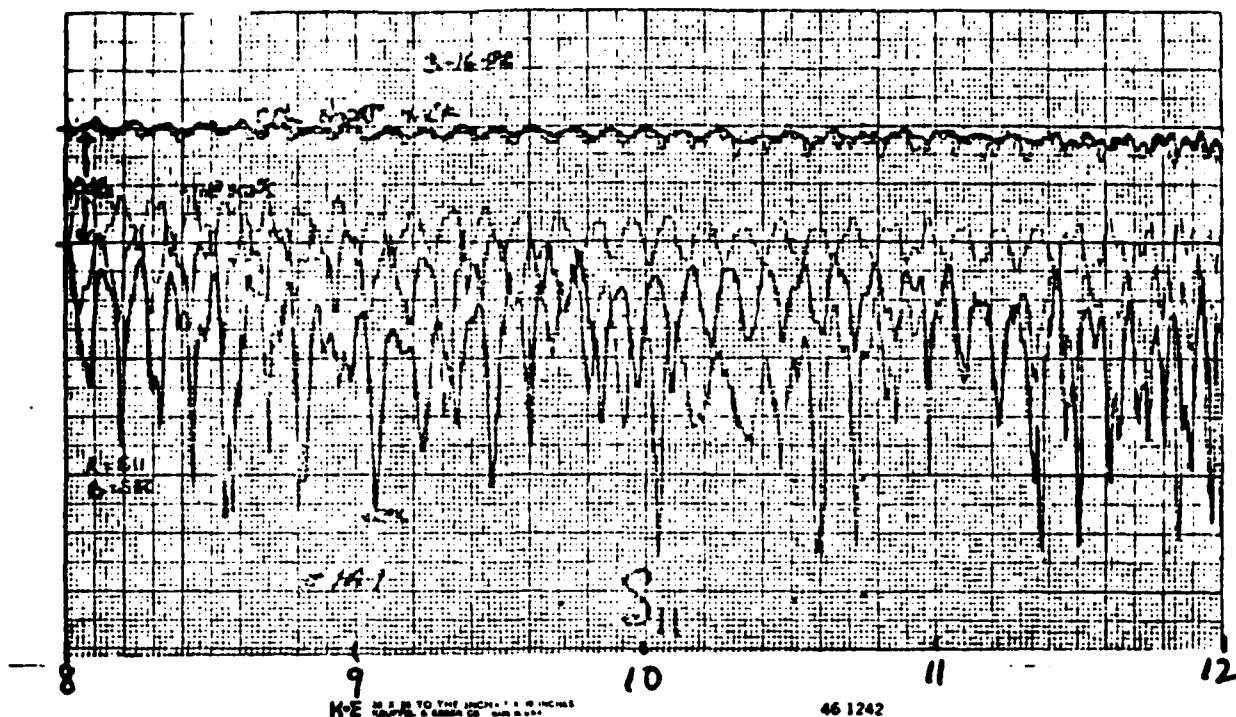


Figure 7. Response of 50Ω tapered coaxial lines on silicon substrate measured at 4K. The upper graph shows  $S_{11}$  (and  $S_{22}$ ) at 4K; the faint curve is  $S_{11}$  at 300K. The lower graph is  $S_{21}$  at 4K.

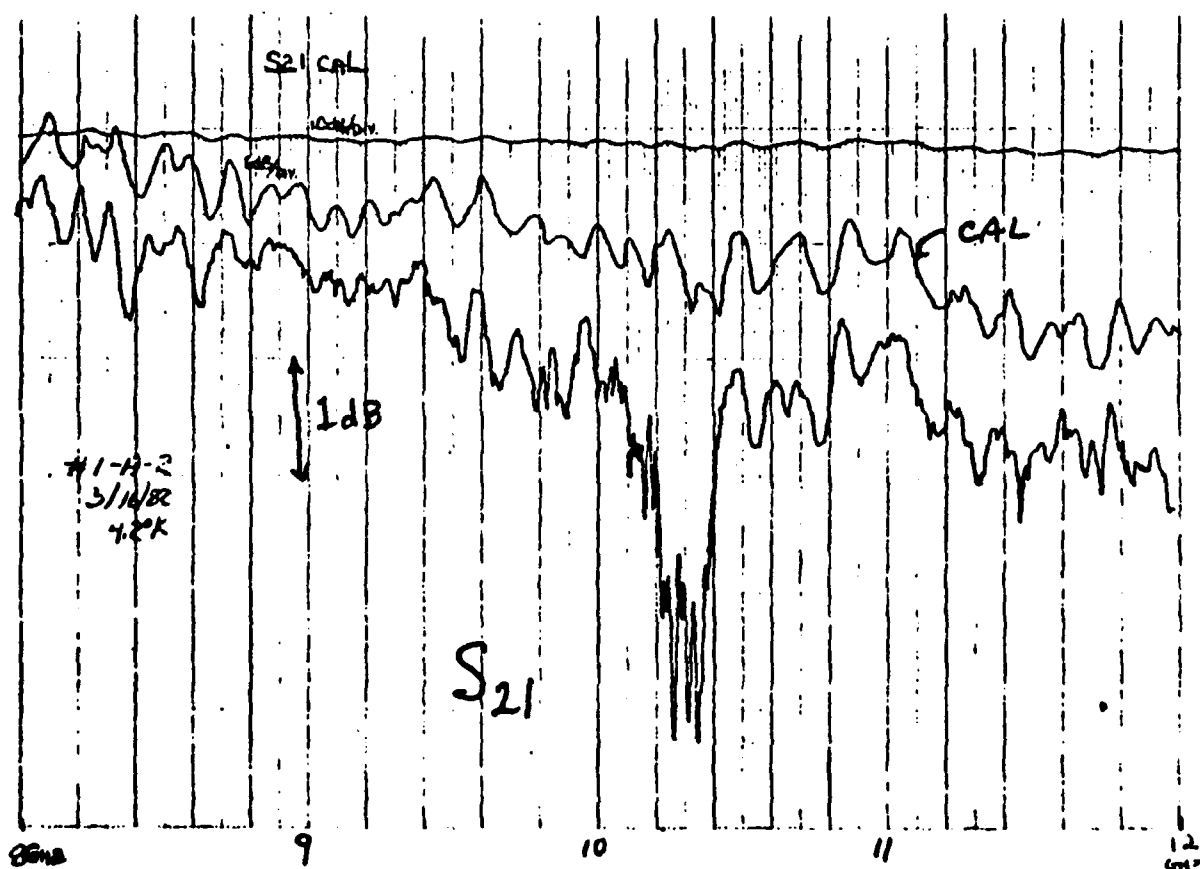
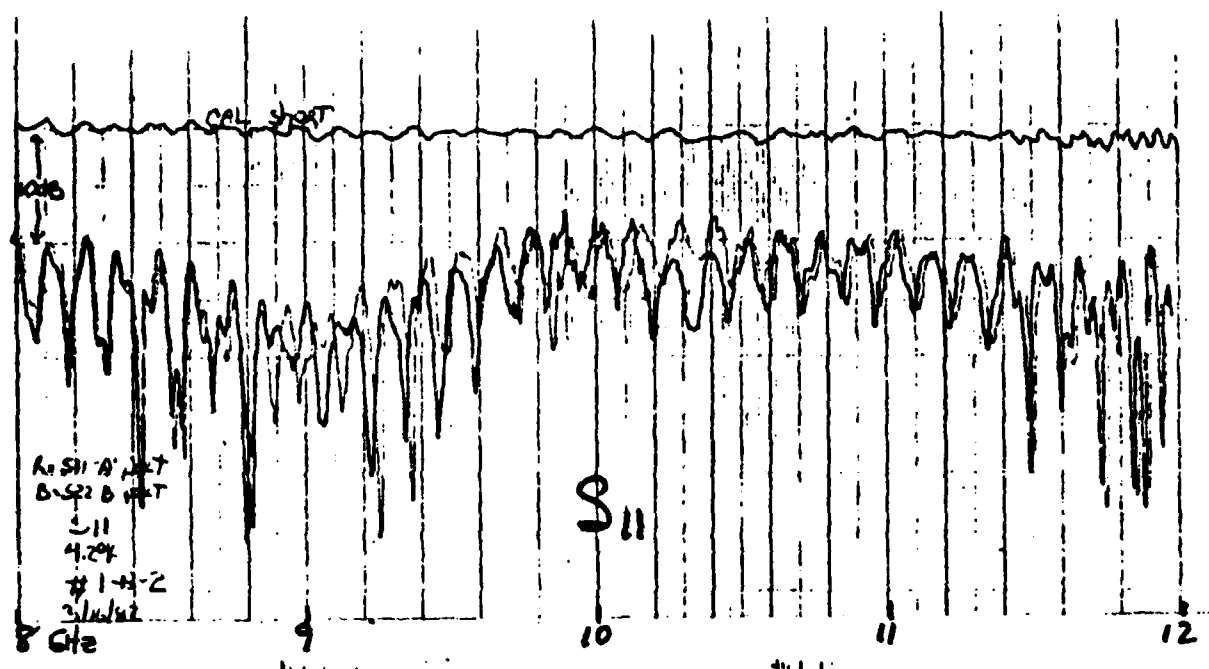


Figure 8. Response of the double-ended 50 $\Omega$  to 1 $\Omega$  transformers measured at 4K. The upper graph is  $S_{11}$  (and  $S_{22}$ ), the lower is  $S_{21}$ .

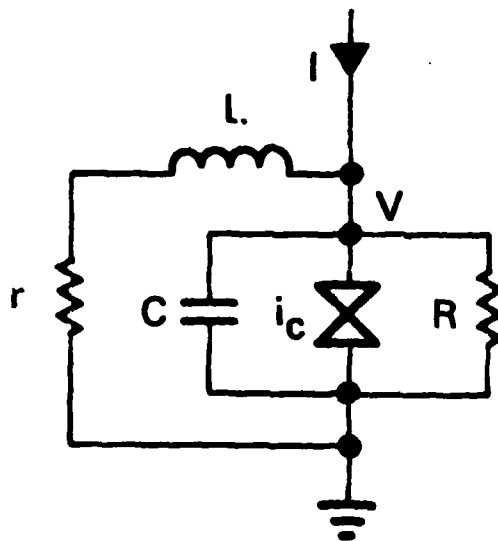


Figure 9. Resistive SQUID VCO.  $L$  is the SQUID inductance and  $r$  the small voltage-biasing resistor.  $R$  is the load resistance which sets the damping.

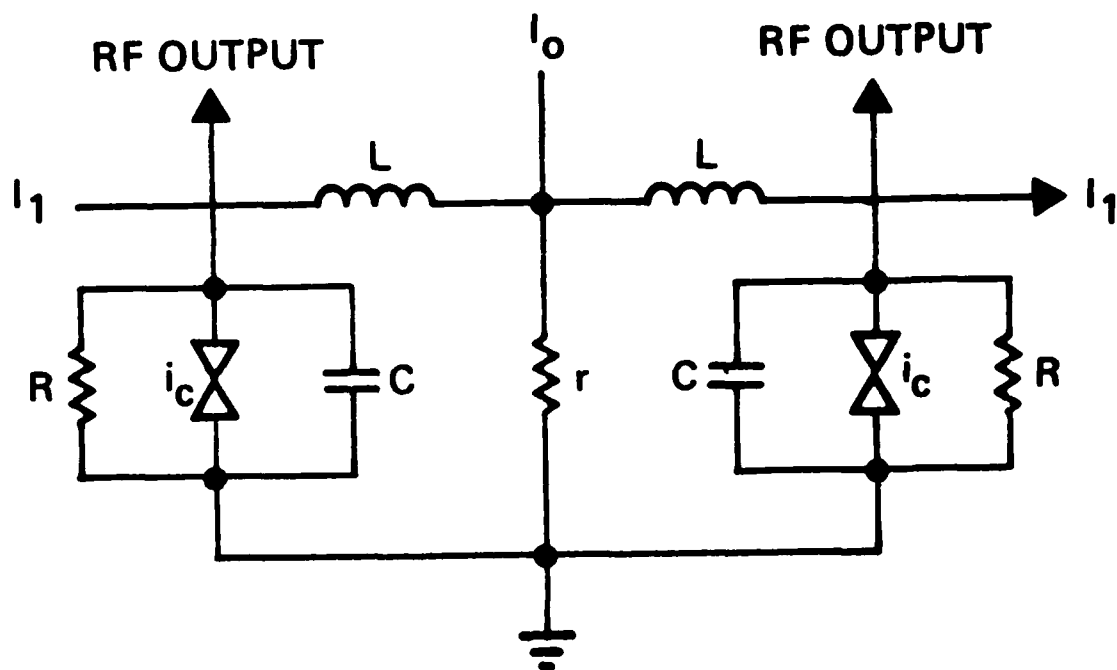


Figure 10. Voltage-clamped dc SQUID. The voltage-clamping resistor is  $r$ , and the load resistance is  $2R$ .



### 3.3 SQUID VCO

#### 3.3.1 Design

The VCO design placed the resistive SQUID at the low impedance end of the  $50\Omega$  to  $1\Omega$  transformer discussed above. Connection to the transformer is with  $1\Omega$  superconducting microstripline. Two devices are placed on each  $1\text{ cm} \times 2\text{ cm}$  chip; four chips are designed on each wafer. The SQUID was designed to utilize the  $\text{Nb}/\text{Nb}_2\text{O}_5/\text{PbBi}$  junctions in use at TRW, with  $\text{PbIn}$  interconnecting lines. Figure 11 shows the composite artwork for the VCO wafer. The structures at the top and bottom of each chip are test junctions and resistors.

The impedance of the SQUID,  $\sqrt{L/C}$ , is set at  $0.3\Omega$ , such that the loading of the  $1\Omega$  transformer will result in  $Q=3.3$ . Design values are chosen to maximize  $\phi_0^2/2L=10^4 kT$  and set  $\beta=1$ , the SQUID resonance frequency equal to  $9\text{GHz}$ , and the Josephson plasma resonance frequency also equal to  $9\text{GHz}$ . Thus  $L = 5\text{pH}$

$$C = 65\text{pF}$$

$$i_c = 67\mu\text{A}$$

$$j_o = 9\text{A}/\text{cm}^2$$

and the junction area is  $50\mu\text{m} \times 14.9\mu\text{m} = 745\mu\text{m}^2$ . The biasing resistance  $r$  is independently set with a design value near  $10^{-3}\Omega$ . This design assumes that the  $1\Omega$  load dominates over the internal quasiparticle resistance.

Because of the nature of the response for  $Q>2$ , we varied the design for each of the four chips on the wafer as follows:

VCO #	L(pH)	C(pF)	Q	$i_c(\mu\text{A})$	$R_d(\Omega)$
1	5	65	3.3	67	---
2	10	32.5	1.7	33.5	---
3	5	65	1.7	67	1
4	2.5	130	1.7	134	0.25

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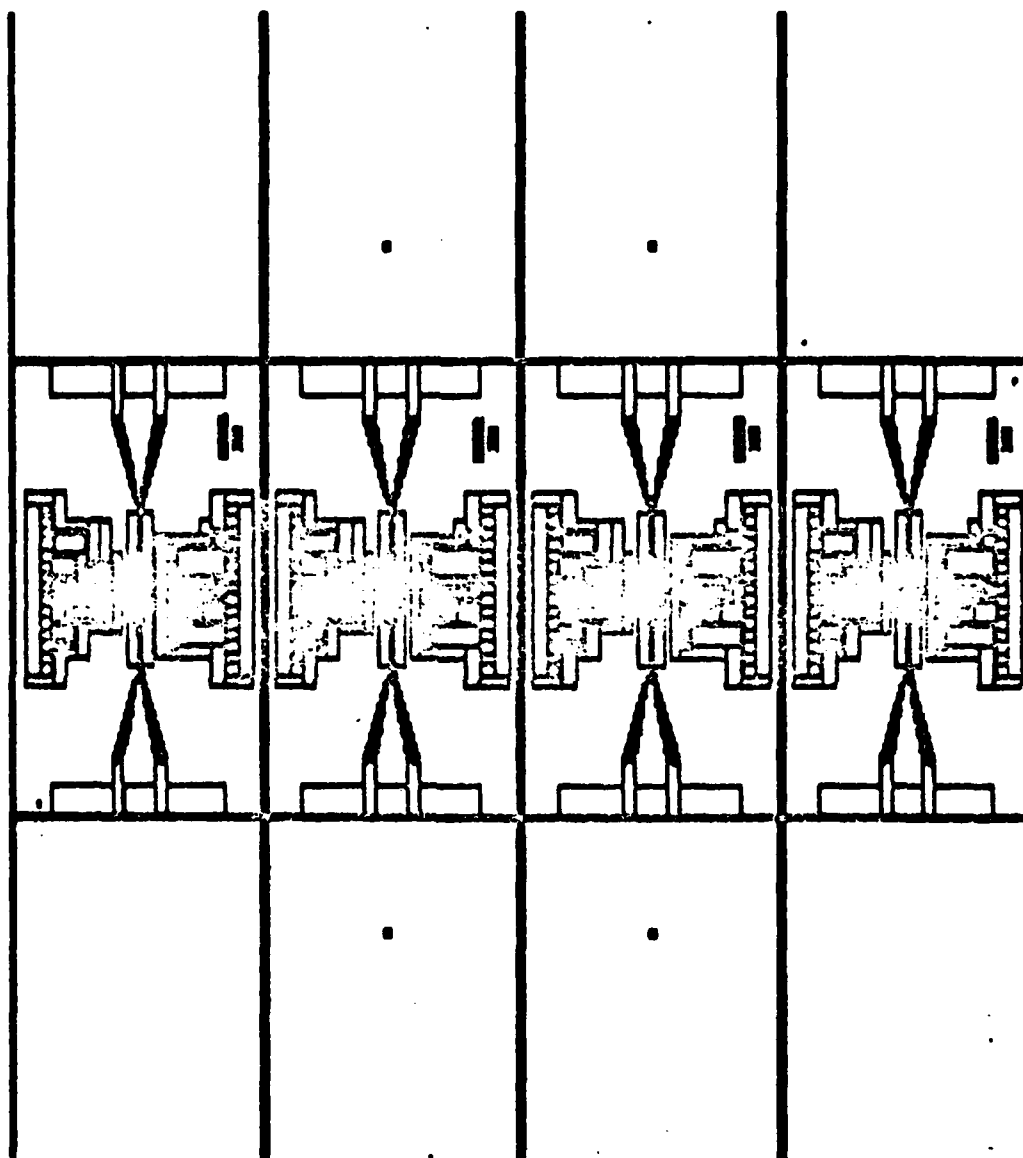


Figure 11a. Composite of seven masks designed for the four VCO circuits. Test junctions and resistors are placed above and below each of the 2 VCO's per chip.

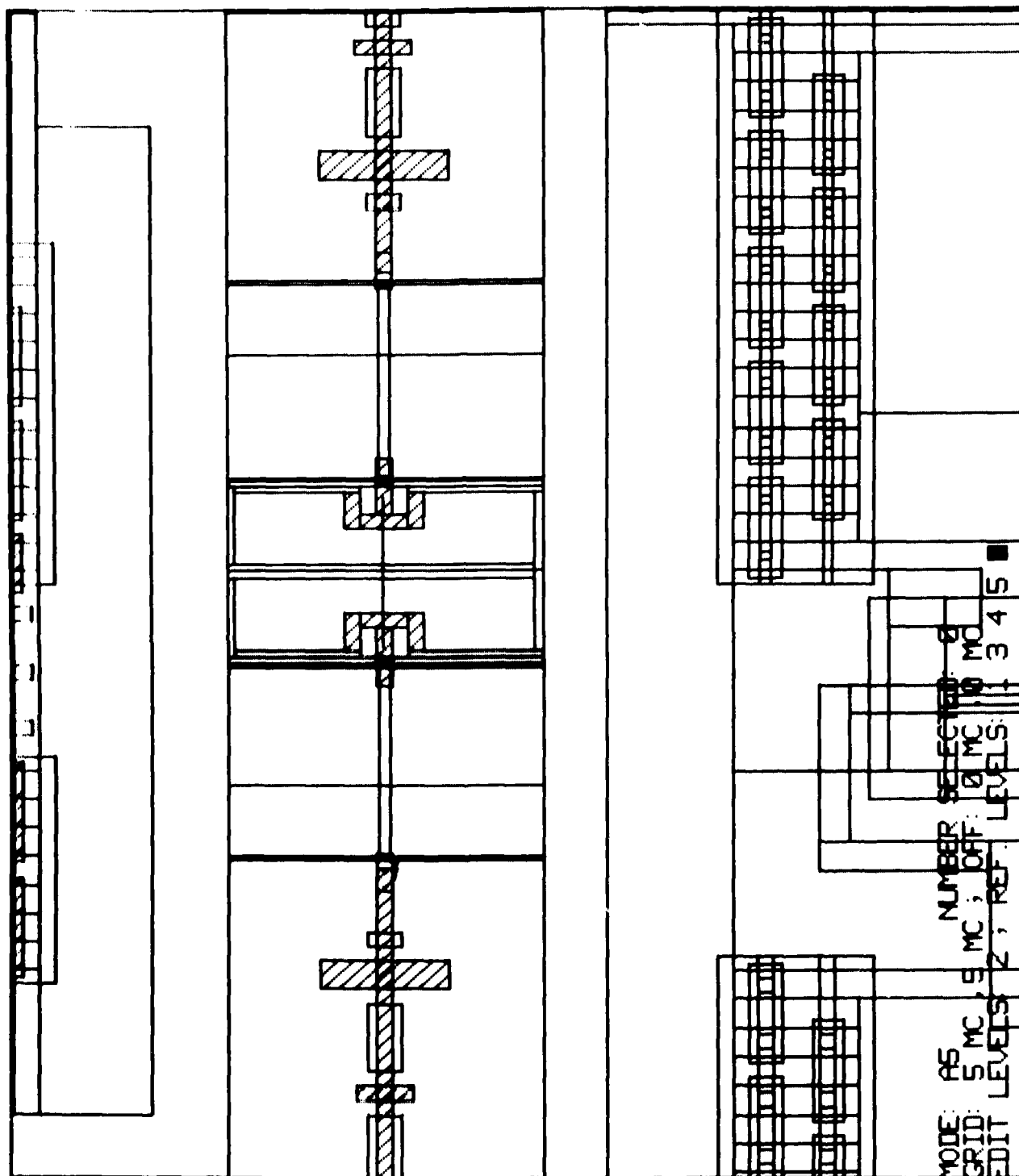


Figure 11b. Composite of mask artwork for the central section of VCO #2.

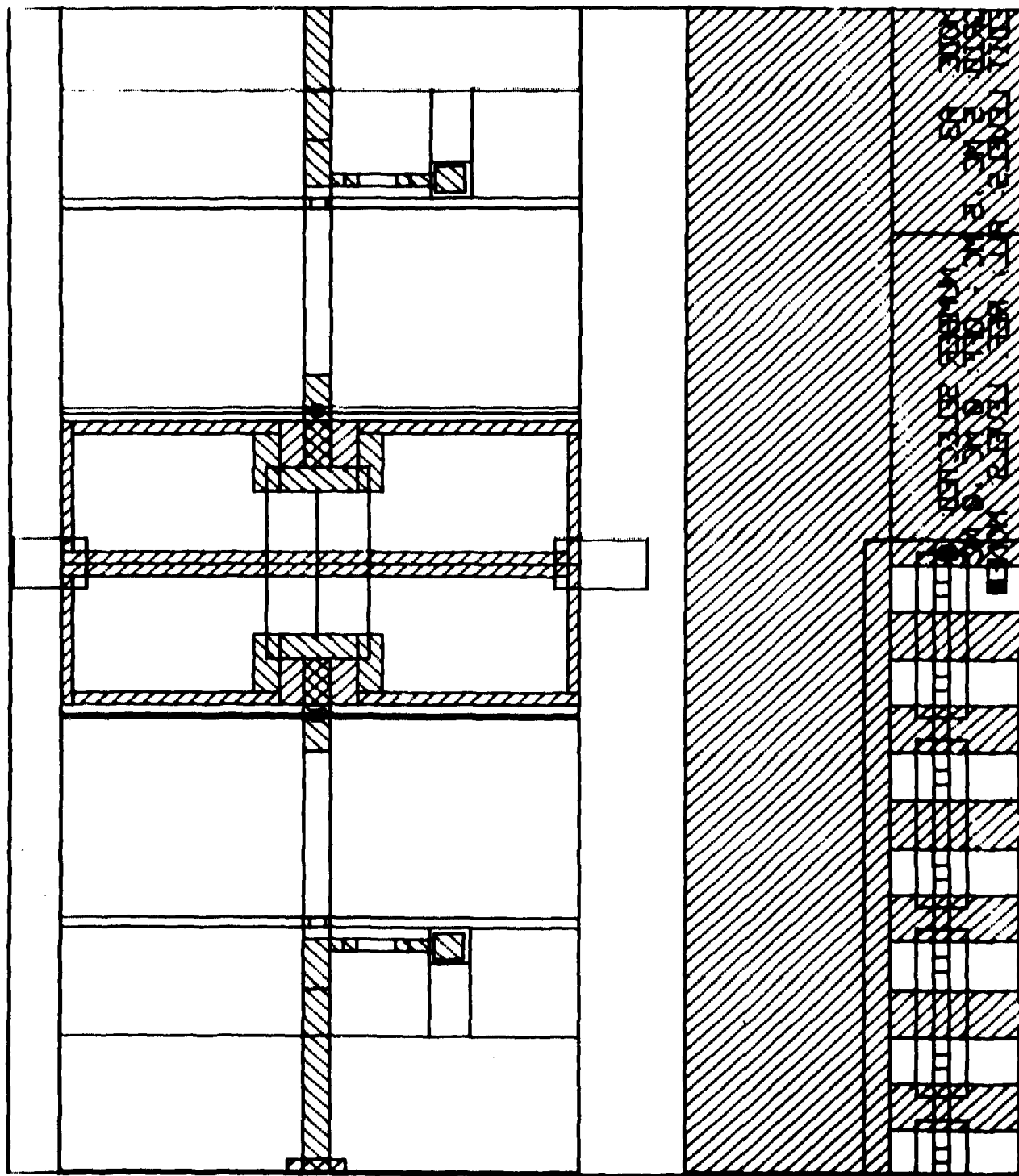


Figure 11c. Composite of mask artwork for the central section of VC0 #3.

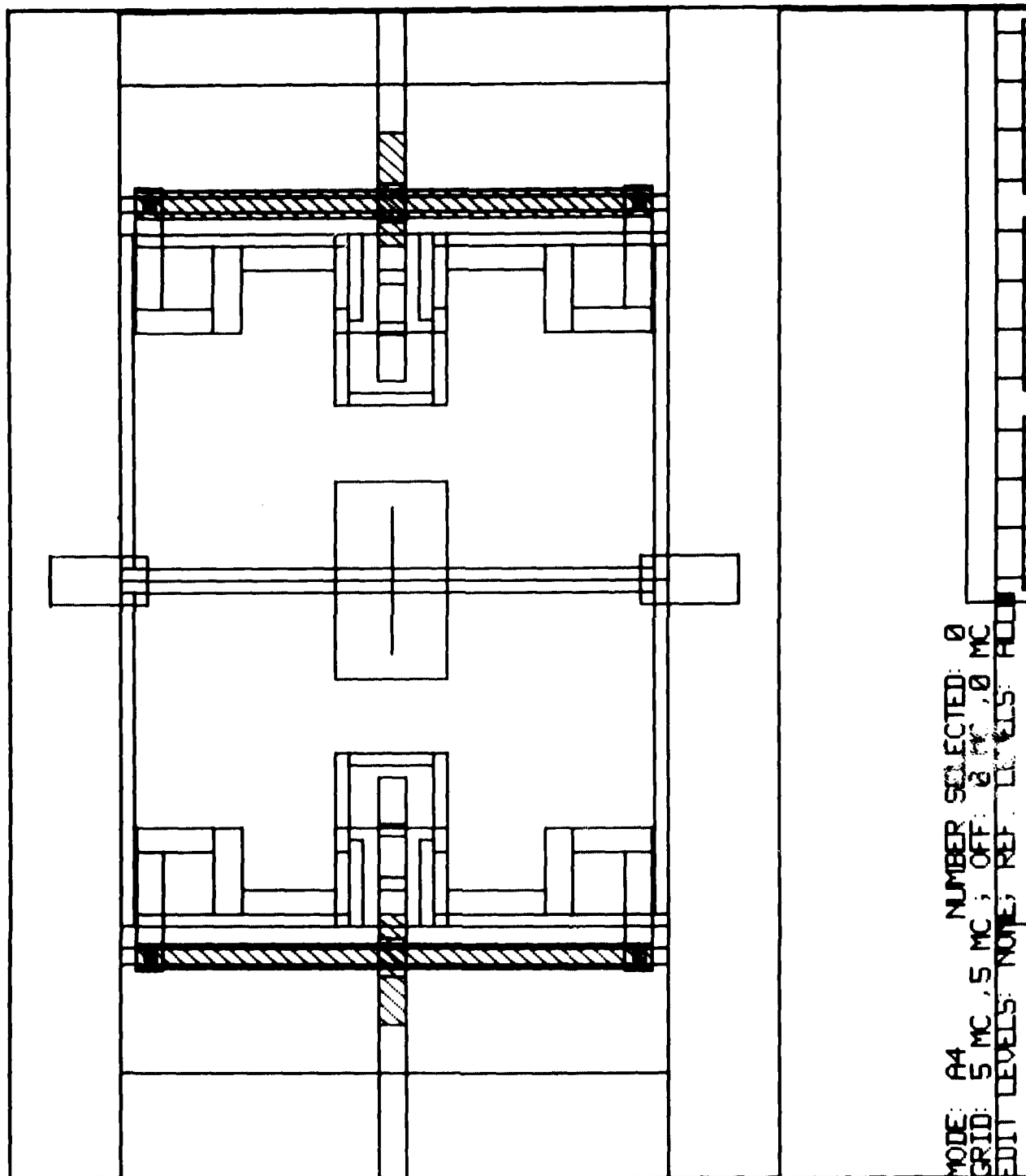


Figure 11d. Composite of the mask artwork for the central section of VCO #4.

$R_d$  is an additional damping resistance placed across the junction to lower the  $Q$  below 2. Although a substantial fraction of the microwave power is lost in the additional damping resistances in VCO 3, 4, they provide broad band damping compared to the transformed load of VCO 1, 2 which is well-defined only over the transformer bandwidth near 9GHz. In VCO 2 the inductance was doubled, halving the flux quantum energy, consequently reducing the available output power and the loaded  $Q$ .

### 3.3.2 Fabrication

The composite VCOs, which include the voltage-clamped resistive SQUIDs and matching transformer, were fabricated on 2" silicon wafers using Nb/Nb<sub>2</sub>O<sub>3</sub>/PbBi junctions, Au resistors, and PbIn interconnection lines. This represents a change from the sapphire substrates originally planned, but permits multiple chip fabrication and easy dicing of the wafer. Also, this method eliminates photoresist problems which occur at the edge of the wafer.

The change from sapphire to silicon substrates produced a problem holding substrates in the vacuum system. Since silicon is more fragile than sapphire, one cannot rigidly clamp the substrate for efficient heat sinking. Vacuum grease is effective at providing efficient heat transfer and mechanical support even for substrates mounted upside down. However, the grease is a potential contaminant for the process which occurs after the vacuum deposition. We have adopted a practice of scraping the grease and cleaning with hot xylene before doing lift-off in acetone. This technique appears manageable although clearly not optimum.

A more common problem was one of multi-layer deposition and processing where good electrical contact is required between layers. Three different vacuum systems were used for this project: an S-Gun sputtering system for Nb deposition, an ion gun system for junction formation, and an e-beam evaporator for other depositions. Only the ion gun system permitted in vacuo cleaning before deposition. The PbBi junction counter-electrode is thermally evaporated in the ion gun system immediately after reactive-ion-oxidation. However, we found significant problems with contacts

when films were deposited in e-beam system. On a temporary basis, we were thermally depositing other metals in the ion gun system, thereby permitting ion cleaning of the surface immediately prior to evaporation. This showed some improvement but was not completely successful, particularly for the alloy films. Also, this multiple use of the ion gun system led to contamination of the system with other materials prior to junction formation.

This problem was addressed more extensively under our IR&D program and we believe it has been resolved successfully. The e-beam system was modified mechanically and electrically to include an rf sputtering stage on which the substrates are mounted. RF sputter cleaning is then performed immediately prior to each new evaporation. Results with this new system are very encouraging. We have fabricated VCO circuits which show satisfactory junctions and connections. Voltage-biasing resistors in the 50-100m $\Omega$  range have been fabricated, one or two orders of magnitude larger than desirable. Figure 12 shows photographs of the SQUID section of the four different VCO chips after fabrication.

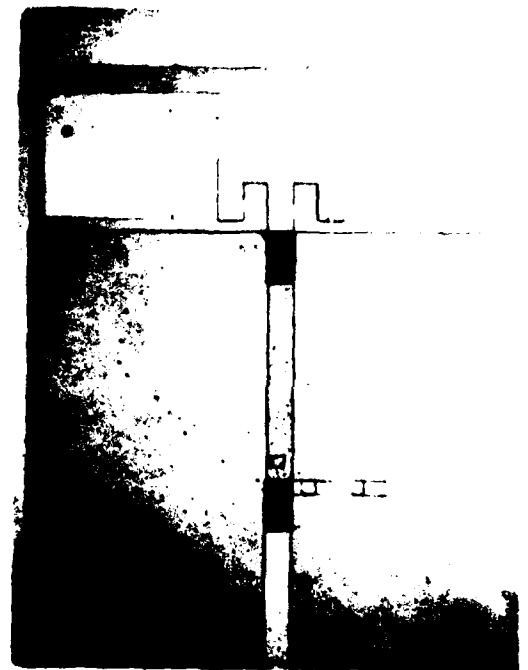
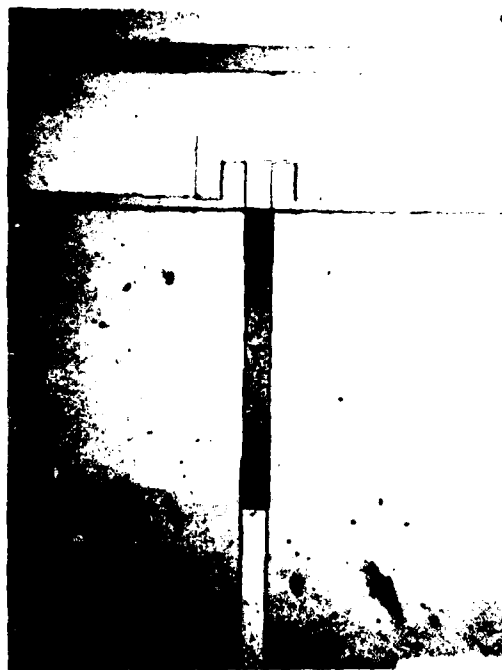
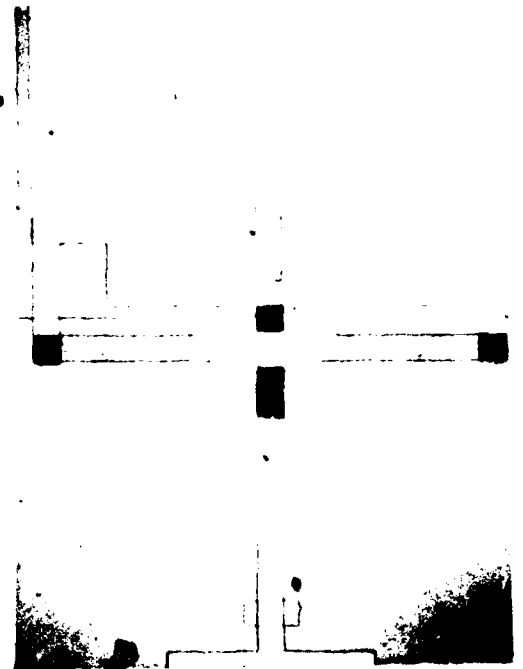
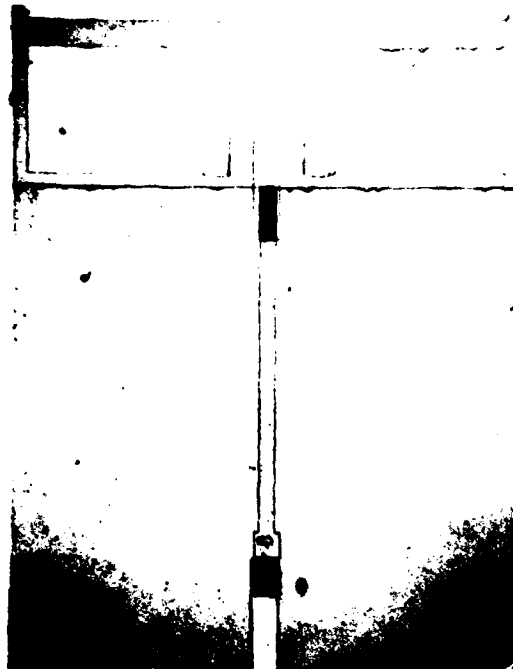


Figure 12. Photographs of the central sections of the four different VCO chips after fabrication.



#### 4. CONCLUSIONS

Measurements show that the SQUID does generate microwave power, but the linewidth is much broader than expected. We believe this is a result of both external noise and internal instabilities of the type identified in 3.2. We are continuing to refine the measurements and SQUID parameters to demonstrate a well-behaved VCO.

## SQUID VOLTAGE-CONTROLLED-OSCILLATOR\*

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## Abstract

We have investigated the SQUID as a voltage-controlled source of microwaves. The low impedance "resistive" SQUID can be a relatively high power ( $\sim 10$ W), tunable, and monochromatic source for both on-chip and off-chip applications. Studies of the time-dependent junction phase and the available power spectra as they vary with such device parameters as loaded  $Q$  and the SQUID  $\Phi = 2\pi\Phi_0/\phi_0$  establish design rules for a well-behaved oscillator. For a VCO  $\Phi < 2$ ; for  $\Phi > 2$  degenerate parametric subharmonic oscillations and chaotic instabilities are observed. Power increase is suggested by the use of voltage-clamped dc SQUIDS and arrays.

## Introduction

The superconducting voltage-controlled-oscillator (VCO) has potential applications as an integrated, on-chip source for such superconducting devices as analog-to-digital converters (sampling clock), rf SQUID magnetometers (excitation source), parametric amplifiers (pump source), quasiparticle mixers (local oscillator), and intracomputer communications. A source of sufficiently high power can be useful as an agile signal generator. This paper describes the SQUID VCO as the implementation of this source.

The impetus for development of microwave/millimeter wave signal generators in superconducting technology has its origin in the Josephson relations which predict a sinusoidal Josephson current under the application of a dc voltage which is linearly related to the Josephson frequency,  $\omega_J = 2\pi V/\Phi_0$ . Unfortunately, problems of impedance and the dynamics of the junction phase have made it difficult to achieve the expected performance.

We argue here that the SQUID and SQUID arrays are the natural forms for a VCO. In order to achieve linear tuning, the source resistance must be small compared to the junction impedance. Such a small resistance connected directly across the junction would short out the Josephson oscillations, greatly reduce the signal voltage across the junction, and no power will be delivered to the load. However, in a resistive SQUID (Fig. 1) the voltage-biasing resistor is isolated from the junction by the SQUID inductance  $L$ . Thus, the dc voltage will be developed across the bias resistor, the ac voltage across the inductance, and the total voltage across the junction.

Tunnel junctions generally have lower internal conductance than microbridges, at least below the energy gap, and are desirable junctions to minimize

internal losses. The shunt capacitance of the tunnel junction, which is frequently considered a serious shunting impedance, can be effectively parallel-tuned by the SQUID inductance. The load resistance, together with the internal junction resistance, will determine the loaded  $Q$  of the VCO.

One expects the instantaneous signal bandwidth to be determined by the Johnson noise across the net (low frequency) resistance as a result of frequency modulation of the oscillator by the thermal voltage fluctuations. This has been predicted and verified for point contact devices<sup>1</sup> as  $\delta f = 4\pi kT/\phi_0^2 = 4.3 \times 10^{-10} RT$  for a biasing resistance  $r$  at temperature  $T$ . This value is independent of frequency to the extent that the bias current is stable and noise-free.

The stable operating mode of a low  $\Phi = 2\pi\Phi_0/\phi_0$  SQUID consists of regular flow of quantized flux  $\Phi_0$ . The internal circulating power in the SQUID is essentially that of changing quantized flux in the inductance  $L$  at the oscillation frequency,  $P_{int} = \Phi_0^2 \omega_J / 4\pi L$ , while the power delivered to the load resistance is  $V^2/R$  where  $V = \Phi_0 \omega_J / 2\pi$ . Thus, the power in the load is  $P_L = P_{int} / Q$  where  $Q = R/\omega L$ . Realistic values of  $L$  approximately  $10^{-11}$  H project to  $\sim 10^{-10}$  W of power near 10GHz, with power increasing linearly with frequency at constant  $Q$ . This relation for  $P_L$  should be valid as  $Q$  decreases until the VCO is sufficiently loaded by  $R$  to reduce the ac voltage. Fundamental to achieving these power levels is small  $L$  and small  $Q$ . This requirement leads directly to load resistance values  $R = Q\omega L$  of the order of  $10^{-1} \Omega$  for  $Q=1$ ,  $L=10^{-11}$  H, and  $f_J \sim 10$ GHz. Proper impedance matching is clearly necessary for both on-chip and off-chip applications. Coherent arrays of SQUIDS which will increase the total impedance can increase the available power as well as simplify the impedance matching.

## Resistive SQUID

We have investigated the voltage-clamped SQUID in two forms: the resistive SQUID (Fig. 1a) and the dual resistive SQUID with common voltage-biasing resistor which we call the voltage-clamped dc SQUID (Fig. 1b). The dynamical response of the SQUID shown in Fig. 1a was simulated numerically to predict the operating characteristics of the SQUID VCO. We envision  $R$  to be the load to which power is delivered. This circuit obeys the relation

$$\ddot{\theta} + (\eta_R + \eta_J)\dot{\theta} + (1 + \eta_J \eta_R + \beta \cos \theta)\dot{\theta} + \eta_J \beta \sin \theta = \dot{\beta}_J + \eta_J \dot{\beta}_J \quad (1)$$

where  $\theta$  is the junction phase,  $\eta_R = \sqrt{L/C}/R\omega^{-1}$ ,  $\eta_J = r/\sqrt{L/C}$ ,  $\beta = 2\pi\Phi_0/\phi_0$ ,  $\beta_J = 2\pi\Phi_0/\phi_0$ , and time is measured in  $1/\omega_J = \sqrt{LC}$ . The voltage across the junction and load resistor  $R$  is  $V = \Phi_0 \omega_J \theta / 2\pi$ . The last term on the right hand side of Eq. (1) is approximately the expected Josephson frequency  $\omega_J/\omega$  for  $1 \gg \eta_J$  and is equivalent to a voltage source  $\eta_J$  in series with  $r$ ; the actual dc voltage across the junction is somewhat lower than  $rI$  and must be determined by either direct measurement or calculation of  $\langle \theta \rangle$ .

The simulations derived  $\theta(t)$  and  $\dot{\theta}(t)$  for selected values of  $\eta_J$ ,  $\eta_R$ ,  $\beta$ , and  $\beta_J$  with  $\dot{\beta}_J = 0$ . The results are essentially independent of  $\eta_J$  for all reasonable values of interest and so  $\eta_J = 10^{-3}$  is used in all calculations reported here. This means that  $I = 10^3 I_c$  and, hence,  $\beta_J = 10^3$  at  $\omega_J = \omega_0$ .

The transient response is dominated by  $\omega_0$ ,  $Q$  and  $\omega_J$ . Figure 2 shows the time evolution of  $\theta$  and  $\dot{\theta}$  for  $\beta=1$ ,  $Q=1$  and 5 for selected values of the voltage bias. For  $Q=1$ , the turn-on transient is short

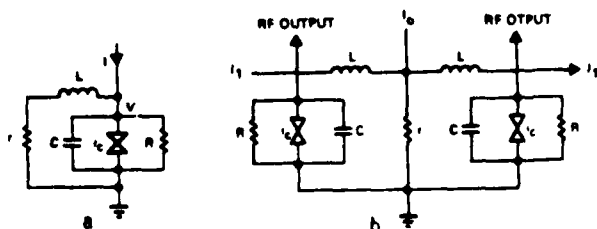


Figure 1. Equivalent circuit of SQUID VCO. (a) Resistive SQUID; (b) Voltage-clamped dc SQUID.

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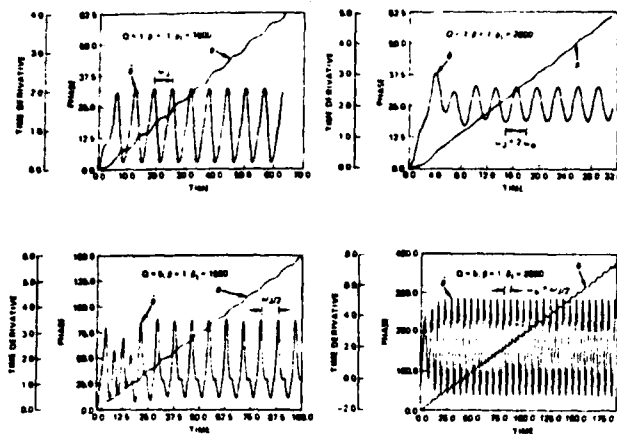


Figure 2. Selected transient response of  $\theta$  and  $\dot{\theta}$  for the resistive SQUID with  $\beta=1$ . Time is in units of  $\omega_0^{-1}$ .

(approximately  $Q/\omega_0$ ) and associated with shock excitation of the resonant LC circuit, the fundamental frequency of the periodic response is  $\omega_J$ , and the periodicity in a smoothly stepped  $\theta$  is  $2\pi$ . If the  $Q$  is raised to 5, the transient response is longer and the resonance frequency becomes further involved with the oscillation. At  $\omega_J/\omega_0=1.5$  ( $\beta_1=1500$ ), there is a strong subharmonic component evident in the time response. At  $\omega_J/\omega_0=2$  ( $\beta_1=2000$ ), the response appears to be totally at  $\omega_0=\omega_J/2$ , while for  $Q=1$  there was no evident subharmonic response. Generally, for either large  $\beta$  or large  $Q$ , the response becomes very complex and even aperiodic. The values required are not 100's, but of the order of 5. Even for smaller values, peculiarities occur somewhere in the spectrum if  $\beta Q > 2$ .

In order to classify and quantify these measurements we have computed the power spectra for selected values of  $Q$  and  $\beta$  as a function of  $\beta_1$ . These were computed over 100 periods of the fundamental frequency of the oscillator after a long time interval after turn on, approximately  $10^3$  cycles. The Fourier amplitudes, including the dc value of  $\langle \theta \rangle$ , were calculated. The zero frequency amplitude was then compared to the fundamental frequency which was first calculated in establishing the time interval for the numerical integration. In general the fundamental frequency is  $\langle \dot{\theta} \rangle = \omega_J/n$ , except for anharmonic cases for which no fundamental frequency was found. Since the Fourier analysis was performed by a Fourier series no spectra were determined for anharmonic cases.

The power delivered to the load resistance  $R$  at any frequency is  $P_i = V_i^2/R$  where  $V_i$  is the Fourier amplitude of the voltage at that frequency. The value computed numerically is  $\langle \dot{\theta} \rangle^2/Q$  which can be converted to physical units according to the relation

$$P_i = \omega_0^2 \phi_0^2 \langle \dot{\theta} \rangle^2 / (2\pi)^2 L Q. \quad (2)$$

Figure 3 shows the dependence of the power delivered at  $\omega_J$  for  $\omega_J=\omega_0$  as a function of  $Q$  for  $\beta=1$ . The power has the expected maximum near  $Q=3$ , decreasing at high  $Q$  as  $Q^{-1}$  and at low  $Q$  because of loading of the SQUID. The maximum value of the power is approximately  $1.124 \phi_0^2 \omega_0^2 / (2\pi)^2 L$ . For design values  $\omega_0=2\pi \times 10^{10}$  and  $L=1\mu H$ , this corresponds to 12.9nW. However, as we have seen, values of  $Q > 2$  lead to some undesirable spectral response. Nevertheless, even for  $Q=1$ , the power is not even diminished by a factor of 2 from the peak value.

Figure 4 shows a compilation of the computed powers of the Josephson oscillations as a function of  $Q$  for  $\beta=1$ . In the regime  $Q < 2$ , the spectra are well

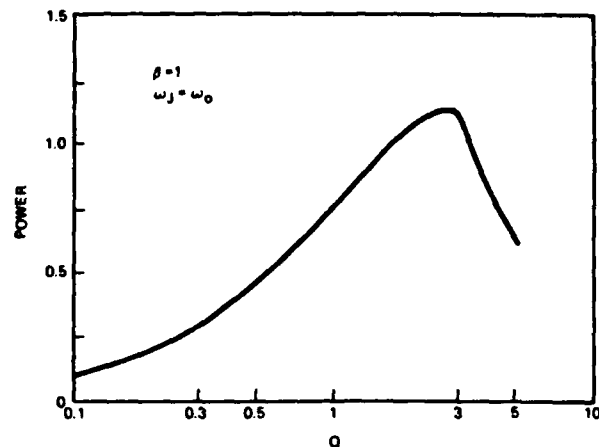


Figure 3.  $Q$ -dependence of the power delivered to the load at  $\omega_J=\omega_0$  for  $\beta=1$  in the resistive SQUID. Power is normalized to  $[\omega_0^2 \phi_0^2 / (2\pi)^2 L]$ .

behaved except for a subharmonic response at  $\omega_J=1.2\omega_0$ ,  $Q=2$ . For  $Q > 2$ , there is a very strong subharmonic response at  $\omega_J=2\omega_0$ , and anharmonic response near  $\omega_J=1.2\omega_0$ . For  $2 < Q < 5$ , the region near  $\omega_J=1.2\omega_0$  exhibits subharmonic behavior which is not plotted.

The strong subharmonic response when  $\omega_J=2\omega_0$ , which occurs at such modest  $Q$ 's as 2.5, suggests degenerate parametric oscillations such as those predicted for the SQUID parametric amplifier<sup>2</sup>. This is also suggestive of the period-doubling effects reported to visit nonlinear systems such as Josephson junctions. However, we also observe odd subharmonic responses for  $Q=5$ ,  $\beta=1$  which do not fall in the period-doubling class. An example of this is shown in Fig. 5 for  $Q=5$ ,  $\beta=1$ , and  $\beta_1=1100$ .

These non-period-doubling subharmonics and the nonharmonic response occur in the general vicinity of  $\omega_J=1.2\omega_0$  for  $Q=5$ ,  $\beta=1$ ; Levinson<sup>4</sup> has recently predicted odd-subharmonic response as the natural form of the instability in highly undamped Josephson junctions. For simulations which were conducted over a wide range of values of  $\omega_J$ , the nonharmonic and odd-harmonic response is very localized in  $\omega_J$ , although we only studied relatively small values of  $Q$ . Inspection of  $\theta(t)$  suggests that  $\theta$  spends relatively more time near  $(2n+1)\pi$  in this regime, compared to  $2n\pi$  for the more well-behaved regime. Since the junction is basically

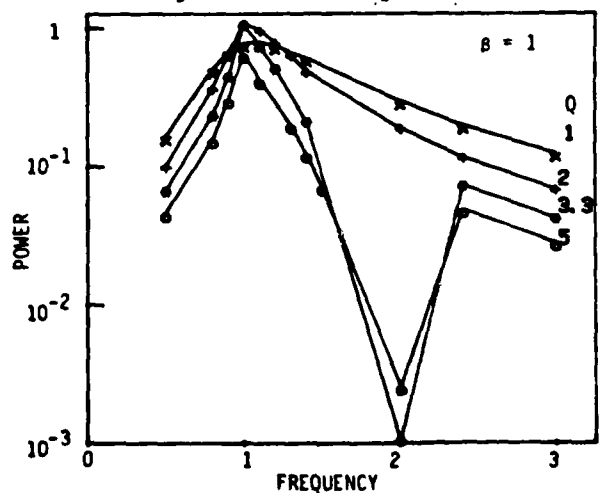


Figure 4. Frequency dependence of the power delivered to the load at  $\omega_J$  for selected  $Q$ -values with  $\beta=1$  in the resistive SQUID. Power is normalized to  $[\omega_0^2 \phi_0^2 / (2\pi)^2 L]$ .

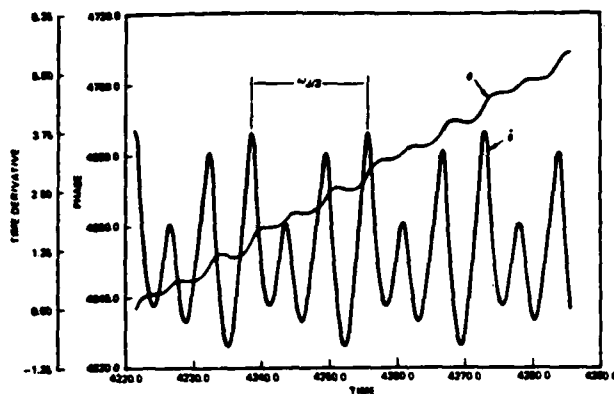


Figure 5. Computed transient response  $\theta$  and  $\dot{\theta}$  for the resistive SQUID with  $\beta=1$ ,  $Q=5$ , and  $\beta_1=1100$ , corresponding to  $\omega_J=1.1$ . Time is measured in  $\omega_0$ .

unstable for principal values  $\pi/2 < \theta < 3\pi/2$ , this may be related to the instability.

The expected effect of increasing  $\beta$  at constant  $Q=1$  is illustrated in the power spectra of Fig. 6. The bracketed symbols are the Josephson frequencies; the symbols connected by straight lines are the predicted harmonics. Not all harmonic spectra are shown. Below  $\beta=\pi$ , all computed points are well behaved with some expected harmonics but no subharmonic response. For  $\beta=\pi$ , we observe subharmonics as we did for large  $Q$  values at  $\beta=1$ . For  $\beta > \pi$ , the time dependent response becomes even more complex and we have not computed spectral powers. Such  $\beta$ -values would be considered outside the normal range for a SQUID and moves toward the non-SQUID Josephson junction for which this analysis indicates very poor quality VCO.

#### Voltage-Clamped DC SQUID

The voltage-clamped dc SQUID of Fig. 1b can be recognized as a combination of two identical resistive SQUIDs with a common biasing resistor  $r$ . This leads to strong coupling of the two VCO's with identical frequencies but relative phases determined by the field current  $I_1$ . Two different modes of the coupled system are essentially a symmetric mode, with the two junctions in parallel, and an antisymmetric mode, with the two junctions in series. Maximum output power and impedance are achieved for the antisymmetric or series mode in which the circulating currents are in the same direction.

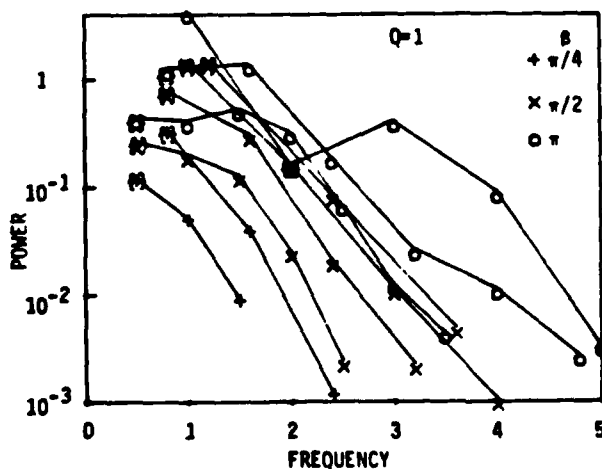


Figure 6. Harmonic spectra for the resistive SQUID with  $Q=1$  and selected  $\beta$ .

We have carried out simulations of the symmetric voltage-clamped dc SQUID shown in Fig. 1b. The new parameters which define the device performance are  $\beta = 2\pi I_1 / \Phi_0$ ,  $\beta_1 = 2\pi I_1 / \Phi_0$ ,  $\theta_+ = (\theta_1 + \theta_2)/2$  and  $\theta_- = (\theta_1 - \theta_2)/2$ .  $\theta_1$  and  $\theta_2$  refer to the phases of the right and left hand junctions, respectively, and we have defined  $\beta$  as  $2\pi I_1 / \Phi_0$ , where  $L$  is the inductance of one-half of the dc SQUID. In a limited number of simulations with  $\beta=1$  and  $\beta_1=0$ , the dc SQUID behaves identically to the resistive rf SQUID with  $\theta_+ = \theta_- = 0$ . The two junctions switch in-phase with one another and the resulting currents in each  $L$  add in the small resistance  $r$ . However, the voltage across the output ( $2L$ ) is identically zero.

For  $\beta_1 = \pi/2$  and  $\beta=1$ , the computation shows that  $\theta_-$  responds at the expected Josephson frequency while  $\theta_+$  responds at the second harmonic. This frequency doubling is a result of the alternate switching of the two junctions in each Josephson period. We have computed the response at  $\beta_1 = \pi/2$ ,  $\beta=1$  for a range of  $Q$  from 1 to 5; the behavior of  $\theta_+$  and  $\theta_-$  is very similar to that observed for  $\theta_+$ ,  $\theta_-$  in the resistive SQUID and will not be reproduced here.

#### SQUID Array VCO

Arrays of SQUIDs, driven coherently, can achieve further increase in power. Three types of arrays are possible: the longitudinal dc SQUID array, the transverse dc SQUID array, and the resistive SQUID bus. We discuss these briefly. Preliminary descriptions of the longitudinal and transverse dc SQUID arrays were given previously in connection with parametric amplification<sup>2</sup>. Figure 7 shows the simple circuits for both the linear dc SQUID arrays, and for a two-dimensional array which combines the two linear arrays. Analysis of the longitudinal flux-flow array has shown that it is unstable with respect to the flux-flow mode which is the one required for the VCO. As Sandell, et al have shown, this type of array can provide series increase in rf impedance although it requires large dc currents because of its parallel nature at dc. Furthermore, in a tunnel junction implementation as compared with microbridges, narrow linewidths will require small biasing resistors as in the voltage-clamped dc SQUID. The transverse array is recognized as a series array of dc SQUID pairs. It provides increased impedance at both rf and dc, offering a reduction in the direct current required. Again, narrow linewidths will require small biasing resistors. It would be undesirable to fabricate large arrays requiring large numbers of carefully matched small resistors.

A method of avoiding the resistor network and producing a high power array is the resistive SQUID bus as shown in Fig. 7. It is an extension of the voltage-clamped dc SQUID to  $N$  resistive SQUIDs with a common voltage-biasing resistor. This could be readily accomplished in a thin film format with a pillbox resistor at the center of a radial array of SQUIDs. These might drive a coaxial line to an off-chip load.

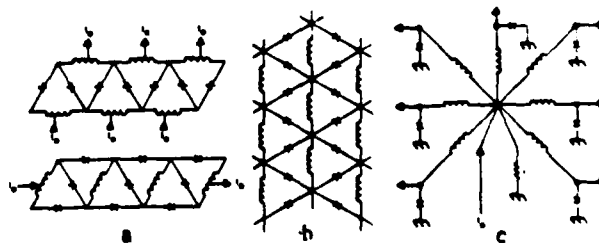


Figure 7. Schematic diagrams of SQUID arrays. (a) Longitudinal (upper) and transverse (lower) flux-flow arrays; (b) Two dimensional array; (c) SQUID bus.

On-chip, the bus can be used to drive many different loads which require coherent input, as in clocking a shift register or driving an array of mixers. If one supplies internal phase shifts of  $\pi$  between adjacent elements (as in the dc SQUID), then one can supply alternate out-of-phase signals. In either case, coherence is guaranteed by the common bias resistor and superconducting phase coherence.

#### Summary

The SQUID is the natural form for signal generation via the Josephson effect. While the bare Josephson junction suffers from both chaotic instabilities and low impedance, its incorporation in a low  $\beta$ , low Q SQUID tunes out the junction capacitance and controls the instability by directly harnessing the periodic nature of the junction phase to the quantum periodicity of the SQUID magnetic flux. This results in an efficient conversion of the Josephson energy to the circulating current in the SQUID inductance and, hence, to external circuitry. The power is maximized by minimizing the inductance, although at further reduction in source impedance. This last problem is alleviated by forming a priori coherent, stable arrays of SQUIDs to increase both the total power and impedance. Such arrays have the voltage-clamped dc SQUID as the basic cell. The design presented makes minimum demands on both lithography and Josephson current density.

Several experimental results have been reported in recent years which pertain to this device. The group at SUNY have studied microbridge VCO's, including pairs and arrays. The most successful results were in the dc SQUID-like arrays reminiscent of Fig. 7a, although the SQUID inductances were very large compared to the values suggested here. Calander and Zappe proposed using a dc SQUID as a tunable oscillator for intracomputer communication in a Josephson processor. They predicted  $\sim 1\mu\text{W}$  of power at 500 GHz. Tuckerman demonstrated this concept by connecting a dc SQUID transmitter with another dc SQUID receiver by a superconducting microstripline. The transmitted microwave signals were readily detected. Calendar, et al. demonstrated a resistive SQUID VCO incidentally to their self-pumped Josephson parametric amplifier. Using a SQUID with  $\beta=2$ ,  $L=3\text{pH}$ ,  $r=3\times 10^{-2}\Omega$ , and a  $1\Omega$  matching transformer, they observed  $0.15\text{nW}$  at 10.4 GHz with a bandwidth of 150 MHz. This linewidth is much greater than the thermal-noise-limited value, 5 MHz, and is reportedly broadened by noise in the bias current. Assuming that the average power is diminished by the same factor as the line broadening, one could have expected as much as  $4.5\text{nW}$  in excellent agreement with the predictions of the model.

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# APPENDIX B

Printout of the computed characteristics of the resistive SQUID VCO. The first line shows the run number,  $\beta$ ,  $Q$ ,  $\beta_1$ , computed fundamental frequency, and subharmonic number relative to the Josephson frequency; the second line is the computed spectral power for the first seven harmonics of the fundamental frequency. Where the frequency and harmonic number are reported as zero, the oscillator is anharmonic.

Run#	$\beta$	$Q$	$\beta_1$	$\omega/\omega_0$	Subharmonic#		
	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$
1	0.78	1.00	500	4.993E-01	1		
	1.215E-01	5.192E-02	9.052E-03	6.037E-04	4.405E-05	4.121E-06	3.816E-07
2	0.78	1.00	800	7.989E-01	1		
	3.369E-01	4.023E-02	1.706E-02	5.912E-05	2.491E-06	1.111E-07	5.115E-09
3	0.78	1.00	1000	9.987E-01	1		
	5.172E-01	2.063E-02	5.123E-04	1.546E-05	4.701E-07	1.446E-08	4.479E-10
4	0.78	1.00	1200	1.199E 00	1		
	4.861E-01	9.236E-03	1.597E-04	2.920E-06	5.434E-09	1.019E-09	1.917E-11
5	0.78	1.00	2000	1.998E 00	1		
	1.885E-01	4.000E-04	1.231E-06	3.279E-09	8.769E-12	2.344E-14	5.812E-17
6	1.57	1.00	500	4.989E-01	1		
	2.686E-01	2.043E-01	1.304E-01	2.566E-02	2.433E-03	6.043E-04	1.489E-04
7	1.57	1.00	800	7.985E-01	1		
	7.978E-01	3.118E-01	2.104E-02	2.274E-03	3.008E-04	3.683E-05	4.331E-06
8	1.57	1.00	1000	9.982E-01	1		
	1.476E 00	1.644E-01	1.150E-02	1.064E-03	9.239E-05	8.231E-06	7.360E-07
9	1.57	1.00	1200	1.198E 00	1		
	1.500E 00	8.592E-02	4.932E-03	2.864E-04	1.682E-05	9.952E-07	5.900E-08
10	1.57	1.00	2000	1.998E 00	1		
	6.933E-01	6.667E-03	6.659E-05	6.709E-07	6.783E-09	6.850E-11	6.794E-13
11	3.14	1.00	500	4.977E-01	1		
	4.494E-01	4.154E-01	5.230E-01	3.286E-01	6.939E-02	1.259E-02	4.442E-03
12	3.14	1.00	800	7.974E-01	1		
	1.273E 00	1.396E 00	1.910E-01	2.640E-02	1.132E-02	2.722E-03	6.296E-04
13	3.14	1.00	1000	9.970E-01	1		

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3.309E 00	5.415E-01	1.225E-01	2.301E-02	4.516E-03	8.823E-04	1.723E-04
14	3.14	1.00	1200	0.000E 00	0	
0.000E 00	0.000E 00	0.000E 00	0.000E 00	0.000E 00	0.000E 00	0.000E 00
15	3.14	1.00	2000	9.984E-01	2	
4.324E 00	1.621E-01	4.091E-01	8.964E-02	3.360E-03	1.999E-03	9.504E-04
16	1.00	0.10	1000	9.900E-01	1	
9.948E-02	2.472E-04	5.929E-07	1.516E-09	1.097E-11	4.057E-12	3.435E-12
17	1.00	0.25	500	4.978E-01	1	
1.950E-01	1.115E-02	5.591E-04	2.694E-05	1.122E-06	1.172E-07	2.789E-08
18	1.00	0.25	1000	9.957E-01	1	
2.428E-01	3.274E-03	3.636E-05	3.335E-07	7.727E-09	1.178E-09	7.541E-10
19	1.00	0.25	2000	1.992E 00	1	
2.185E-01	4.641E-04	9.824E-07	2.314E-08	1.016E-08	7.187E-09	5.416E-09
20	1.00	0.25	3000	2.988E 00	1	
1.734E-01	9.857E-05	6.609E-08	2.313E-09	1.207E-09	9.153E-10	6.811E-10
21	1.00	0.30	1000	9.963E-01	1	
2.881E-01	5.249E-03	7.395E-05	5.814E-07	1.934E-08	1.904E-09	9.791E-10
22	1.00	0.50	1000	9.974E-01	1	
4.555E-01	1.690E-02	4.041E-04	1.040E-05	4.830E-07	8.373E-08	4.168E-08
23	1.00	1.00	500	4.992E-01	1	
1.660E-01	9.371E-02	2.621E-01	2.561E-03	2.436E-01	3.104E-05	4.096E-06
24	1.00	1.00	800	7.998E-01	1	
5.030E-01	8.673E-02	3.570E-03	2.270E-04	1.510E-05	1.013E-06	6.715E-08
25	1.00	1.00	900	8.987E-01	1	
6.640E-01	6.425E-02	2.416E-03	1.267E-04	7.497E-06	4.143E-07	2.311E-08
26	1.00	1.00	1000	9.986E-01	1	
7.694E-01	4.550E-02	1.675E-03	7.656E-05	3.474E-06	1.593E-07	7.367E-09
27	1.00	1.00	1100	1.099E 00	1	
7.857E-01	3.149E-02	1.029E-03	3.704E-05	1.350E-06	4.969E-08	1.837E-09
28	1.00	1.00	1200	1.198E 00	1	
7.423E-01	2.146E-02	5.723E-04	1.607E-05	4.583E-07	1.316E-08	3.791E-10
29	1.00	1.00	1400	1.398E 00	1	
6.024E-01	9.828E-02	1.015E-04	2.723E-06	4.646E-08	7.962E-10	1.367E-11

30	1.00	1.00	2000	1.998E 00	1		
2.966E-01	1.202E-03	5.073E-06		2.166E-03	9.276E-11	3.937E-13	1.471E-15
31	1.00	1.00	2400	2.398E 00	1		
1.992E-01	3.882E-04	7.841E-07		1.597E-09	3.272E-12	7.205E-15	4.827E-17
32	1.00	1.00	3000	2.997E 00	1		
1.226E-01	9.711E-05	7.909E-08		6.527E-11	6.371E-14	5.032E-12	1.659E-16
33	1.00	1.50	500	4.994E-01	1		
1.248E-01	1.117E-01	4.801E-02		3.853E-03	3.287E-04	5.348E-05	7.592E-06
34	1.00	1.50	800	7.991E-01	1		
4.447E-01	1.182E-01	3.078E-03		2.755E-04	2.120E-05	1.733E-06	1.266E-07
35	1.00	1.50	900	8.990E-01	1		
7.093E-01	8.340E-02	2.555E-03		2.120E-04	1.432E-05	9.526E-07	6.719E-08
36	1.00	1.50	1000	9.988E-01	1		
9.642E-01	5.370E-02	2.812E-03		1.723E-04	1.074E-05	6.825E-07	4.350E-08
37	1.00	1.50	1100	1.099E 00	1		
9.503E-01	3.575E-02	1.312E-03		8.927E-05	4.532E-06	2.373E-07	1.228E-08
38	1.00	1.50	1200	1.199E 00	1		
8.248E-01	2.367E-02	9.123E-04		3.468E-05	1.052E-06	5.289E-08	2.069E-09
39	1.00	1.50	1300	1.299E 00	1		
6.971E-01	1.535E-02	4.352E-04		1.259E-05	3.664E-07	1.068E-08	3.107E-10
40	1.00	1.50	1400	1.399E 00	1		
5.862E-01	9.923E-03	2.119E-04		4.568E-06	9.944E-08	2.167E-09	4.719E-11
41	1.00	1.50	2000	1.999E 00	1		
2.354E-01	9.852E-04	4.693E-05		2.248E-08	1.075E-10	4.840E-13	1.175E-15
42	1.00	1.50	2400	2.398E 00	1		
1.490E-01	2.970E-04	6.405E-07		1.426E-09	3.118E-12	6.130E-15	3.675E-19
43	1.00	1.50	3000	2.998E 00	1		
8.759E-02	7.039E-05	6.001E-08		5.131E-11	4.190E-14	6.238E-18	7.092E-19
44	1.00	1.75	2000	1.999E 00	1		
2.102E-01	8.858E-04	4.332E-06		2.140E-08	1.057E-10	5.150E-13	2.137E-15
45	1.00	2.00	500	4.995E-01	1		
1.007E-01	1.137E-01	6.312E-02		4.135E-03	3.176E-04	6.442E-05	8.002E-06
46	1.00	2.00	800	7.993E-01	1		
3.669E-01	1.270E-01	2.032E-03		2.489E-04	1.280E-05	1.775E-06	1.188E-07



47	1.00	2.00	900	8.991E-01	1		
	6.487E-01	8.875E-02	1.899E-03	2.007E-04	1.490E-05	9.597E-07	7.628E-08
48	1.00	2.00	1000	9.989E-01	1		
	1.071E 00	4.892E-02	3.798E-03	2.529E-04	1.897E-05	1.479E-06	1.119E-07
49	1.00	2.00	1100	1.099E 00	1		
	9.621E-01	3.313E-02	2.212E-03	1.293E-04	7.907E-06	4.914E-07	3.027E-08
50	1.00	2.00	1200	1.199E 00	1		
	7.811E-01	2.195E-02	1.013E-03	4.492E-05	2.031E-06	9.177E-08	4.138E-09
51	1.00	2.00	1300	6.495E-01	2		
	2.677E-09	6.342E-01	3.706E-10	1.399E-02	3.530E-11	4.606E-04	2.671E-12
52	1.00	2.00	1400	6.996E-01	2		
	4.641E-02	4.846E-01	5.459E-03	7.638E-03	3.530E-04	1.648E-04	1.531E-05
53	1.00	2.00	2000	9.995E-01	2		
	1.352E-06	1.890E-01	5.227E-08	7.999E-04	8.518E-10	3.984E-06	2.237E-11
54	1.00	2.00	2400	2.399E 00	1		
	1.167E-01	2.345E-04	5.274E-07	1.195E-09	2.712E-12	6.214E-15	2.180E-17
55	1.00	2.00	3000	2.998E 00	1		
	6.737E-02	5.441E-05	4.472E-07	4.097E-11	3.013E-14	3.603E-19	1.323E-17
56	1.00	2.25	2000	9.995E-01	2		
	4.572E-01	1.021E-01	1.298E-02	3.144E-04	9.165E-05	7.955E-06	5.421E-07
57	1.00	2.50	500	4.995E-01	1		
	8.473E-02	1.008E-01	7.005E-02	3.806E-03	2.721E-04	6.833E-05	6.539E-06
58	1.00	2.50	800	7.994E-01	1		
	3.045E-01	1.243E-01	1.364E-02	2.122E-04	1.404E-05	1.619E-06	9.149E-08
59	1.00	2.50	900	8.993E-01	1		
	5.638E-01	8.772E-02	1.292E-03	1.607E-04	1.300E-05	7.306E-07	6.398E-08
60	1.00	2.50	1000	9.990E-01	1		
	1.124E 00	3.991E-02	4.659E-03	3.138E-04	2.681E-05	2.404E-06	2.038E-07
61	1.00	2.50	1100	1.099E 00	1		
	8.890E-01	2.963E-02	2.219E-03	1.449E-04	9.807E-06	6.609E-07	4.448E-08
62	1.00	2.50	1200	5.996E-01	2		
	1.442E-02	6.861E-01	1.783E-03	1.862E-02	2.928E-04	9.096E-04	2.558E-05

63	1.00	2.50	1300	6.456E-01	2		
	9.231E-02	4.762E-01	1.040E-02	9.082E-03	9.778E-04	2.832E-04	5.693E-05
64	1.00	2.50	1400	6.996E-01	2		
	1.744E-01	3.254E-01	1.594E-02	4.636E-03	9.205E-04	1.224E-04	3.601E-05
65	1.00	2.50	2000	9.995E-01	2		
	9.426E-01	4.213E-02	1.816E-02	1.428E-03	8.202E-05	1.830E-05	2.253E-06
66	1.00	2.50	2400	2.999E 00	1		
	9.531E-02	1.922E-04	4.384E-07	1.008E-09	2.333E-12	5.899E-15	4.805E-17
67	1.00	2.50	3000	2.999E 00	1		
	5.454E-02	4.415E-05	3.859E-08	3.390E-11	2.374E-14	2.681E-17	2.059E-18
68	1.00	3.00	1000	9.990E-01	1		
	1.124E 00	3.054E-02	5.238E-03	4.045E-04	3.456E-05	3.788E-06	3.594E-07
69	1.00	3.00	1100	1.099E 00	1		
	7.966E-01	2.650E-02	2.059E-03	1.445E-04	1.028E-05	7.267E-07	5.126E-08
70	1.00	3.00	1200	5.997E-01	2		
	5.985E-02	5.636E-01	5.706E-03	1.402E-02	1.006E-03	6.915E-04	8.586E-05
71	1.00	2.00	1300	6.497E-01	2		
	1.313E-01	3.738E-01	1.223E-02	7.022E 03	1.163E-03	2.468E-04	6.650E-05
72	1.00	3.00	1400	6.997E-01	2		
	2.067E-01	2.443E-01	1.615E-02	3.879E-03	9.236E-04	1.321E-04	3.637E-05
73	1.00	3.00	2000	9.995E-01	2		
	1.496E 00	5.686E-03	1.455E-02	3.404E-03	2.051E-04	2.020E-05	4.254E-06
74	1.00	3.00	2400	2.999E 00	1		
	8.033E-02	1.623E-04	3.721E-07	3.635E-13	1.052E-12	4.362E-15	6.964E-18
75	1.00	3.00	3000	2.999E 00	1		
	4.575E-02	3.707E-05	3.256E-08	2.889E-11	2.876E-14	1.451E-16	3.204E-17
76	1.00	3.30	500	4.996E-01	1		
	6.735E-02	9.665E-02	7.223E-03	2.889E-03	1.958E-04	6.801E-05	3.832E-06
77	1.00	3.30	800	7.995E-01	1		
	2.356E-01	1.121E-01	9.804E-04	1.647E-04	9.301E 06	1.349E-06	5.520E-08
78	1.00	3.30	900	8.994E-01	1		
	4.496E-01	8.035E-02	7.290E-04	1.060E-04	1.016E-05	4.426E-06	4.215E-08
79	1.00	3.30	1000	9.991E-01	1		
	1.054E 00	3.537E-02	4.534E-03	2.695E-04	2.189E-05	2.356E-06	1.919E-07

80	1.00	3.30	1100	5.497E-01	2		
6.210E-03	7.405E-01	3.157E-04	2.422E-02	1.572E-04	1.912E-03	1.879E-05	
81	1.00	3.30	1200	5.997E-01	2		
7.361E-02	5.091E-01	6.169E-03	1.237E-02	1.141E-03	6.292E-04	9.703E-05	
82	1.00	3.30	1400	6.997E-01	2		
2.102E-01	2.122E-01	1.534E-02	3.528E-03	8.794E-04	1.344E-04	3.533E-05	
83	1.00	3.30	2000	9.995E-01	2		
1.677E-00	1.052E-03	1.122E-02	3.918E-03	4.523E-04	3.031E-05	4.184E-06	
84	1.00	3.30	2400	2.999E-00	1		
7.336E-02	1.493E-04	1.421E-07	7.950E-10	1.658E-12	5.055E-15	6.074E-17	
85	1.00	3.30	3000	2.999E-00	1		
4.170E-02	3.380E-05	2.973E-08	2.636E-11	2.497E-14	7.334E-17	1.121E-17	
86	1.00	5.00	500	4.997E-01	1		
4.627E-02	7.318E-02	6.408E-02	1.410E-03	1.020E-04	5.983E-05	1.268E-06	
87	1.00	5.00	800	7.997E-01	1		
1.570E-01	8.590E-02	3.742E-04	1.079E-04	4.070E-05	9.593E-07	2.060E-08	
88	1.00	5.00	900	8.996E-01	1		
3.055E-01	6.245E-02	3.071E-04	5.070E-05	6.564E-06	1.667E-07	2.168E-08	
89	1.00	5.00	1000	9.995E-01	1		
6.492E-01	4.328E-02	1.775E-03	8.636E-05	7.057E-06	6.227E-07	3.834E-08	
90	1.00	5.00	1100	1.833E-01	6		
2.704E-07	9.691E-03	2.107E-06	1.322E-01	1.432E-04	4.218E-01	3.834E-07	
91	1.00	5.00	1100	3.666E-01	3		
9.690E-03	1.322E-01	4.218E-01	3.160E-03	3.700E-03	1.059E-02	1.191E-04	
92	1.00	5.00	1150	0.000E-00	0		
0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	
93	1.00	5.00	1200	0.000E-00	0		
0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	
94	1.00	5.00	1250	0.000E-00	0		
0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	0.000E-00	
95	1.00	5.00	1300	1.624E-01	8		
9.229E-07	1.800E-04	1.934E-05	1.023E-01	8.447E-05	3.470E-02	4.757E-06	

96	1.00	5.00	1300	3.250E-01	4		
8.010E-04	1.020E-01	3.457E-02	1.998E-01	7.103E-04	6.746E-03	1.336E-03	
97	1.00	5.00	1400	3.499E-01	4		
5.351E-05	1.599E-01	2.534E-02	1.228E-01	4.912E-04	9.375E-03	7.533E-04	
98	1.00	5.00	1500	7.496E-01	2		
2.426E-01	7.130E-02	1.192E-02	2.003E-03	4.446E-04	7.917E-05	1.543E-05	
99	1.00	5.00	2000	9.996E-01	2		
1.840E-00	2.588E-03	2.490E-03	3.572E-03	6.171E-04	1.015E-04	7.962E-06	
100	1.00	5.00	2400	2.400E-00	1		
4.901E-02	9.928E-05	2.308E-07	5.408E-10	1.076E-12	3.333E-15	3.026E-17	
101	1.00	5.00	3000	2.999E-00	1		
2.771E-02	2.249E-05	1.986E-08	1.753E-11	1.382E-14	2.648E-18	1.405E-17	

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